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Magnetized Bianchi Type-VI_0 String Cosmological Model for Anti-Stiff Fluid in General Relativity

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Abstract

In this paper, we have investigated magnetized Bianchi type-VI_0 string cosmological model for anti-stiff fluid in general relativity. We assume that F_{23} is the only survival component of electromagnetic field tensor F_{ij}. For the absolute determination of the model, we assume the expansion in the model is proportional to the shear. The common measures of the Einstein's field equations for the cosmological model have been acquired under the assumption of anti-stiff fluid *i.e.* $p + \rho = 0$, where ρ and p are the rest energy density and the pressure of the fluid, respectively. The physical and geometrical outcomes of the model in the presence of magnetic field are discussed.

Key words : Bianchi type-VI_0, Anti-stiff fluid, Magnetic field, Cosmology (85A40).

1. Introduction

The present day universe is satisfactorily mentioned by homogeneous and isotropic models given by the Robertson-Walker line-elements. It is still challenging problem for us to know actual physical condition at early phase of formation of the universe. Bianchi type models have been widely studied in the framework of general relativity in the search for a realistic picture of the universe in its early stages.

A fundamental feature of the general theory of relativity is that by combining Einstein's field equations with the doubly contracted Riemannian Bianchi identity. Barrow[5] has explored Bianchi type VI_0 universes give a better explication of some of the cosmological problems like primordial helium abundance and they also isotropize in a special sense. Letelier[8, 9] and Stachel[13] have developed the general relativistic treatment of strings.

It is interesting to note that magnetic field existent in galactic and inter galactic spaces and plays to stimulate role at cosmological dimension. The perusal of magnetic field in the matter distribution is of considerable interest as it provides a dominant way to be aware the initial phases of cosmic evolution. The phenomena of magnetic fields on cosmic scale are well-ordered fact today.

A cosmological model containing a global magnetic field is inevitably anisotropic. An understanding of the effect of a magnetic field upon the dynamics of the universe is necessary during early and late time evolution of the universe. As a natural consequence, we should comprise magnetic fields in the energy momentum tensor of the early universe. The importance of the magnetic field for various astrophysical phenomena has been studied in many papers.

Chakraborty[6] has obtained a class of solutions for string cosmology in the context of Bianchi type VI_0 space-time. Tikekar and Patel[14] have investigated some exact solutions for Bianchi type VI_0 cosmology with and without magnetic field. Bali and Upadhaya[4] have investigated LRS Bianchi type I string dust magnetized cosmological models. Amirhaschi[1] has derived string cosmology in Bianchi type-VI_0 dusty universe with electromagnetic field. Bali and Pareek[3] have presented massive string of Bianchi type III cosmological model for perfect fluid distribution in the presence of magnetic field.

Pradhan and Bali[10] have investigated Bianchi type VI_0 string cosmological models in the presence and absence of magnetic field. Pradhan and Lata[11] have investigated Bianchi type-VI_0 bulk viscous string cosmological models with magnetic field. Pradhan *et al.*[12] have derived generation of bulk viscous fluid massive string cosmological models with electromagnetic field in Bianchi type-VI_0 space-time.

Tyagi and Chhajed[15] have investigated homogeneous anisotropic Bianchi type IX cosmological model for perfect fluid distribution with electro-magnetic field. Bali and Bola[2] have investigated Bianchi type- VI_0 massive string cosmological models with magnetic field and time dependent vacuum energy density in general relativity.

Tyagi *et al*.[16] have investigated Magnetized Bianchi type- VI_0 cosmological model for barotropic fluid distribution with variable magnetic permeability and dark energy. Chhajed *et al*.[7] have investigated Magnetized Bianchi type III string cosmological model for anti-stiff fluid in general relativity. Motivated by the above research work, in this paper, we have investigated magnetized Bianchi type-VI_0 string cosmological model for anti-stiff fluid in general relativity. The physical and geometrical virtues of the model in the presence of magnetic field are discussed.

2. The Metric and field equations :

We consider Bianchi type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2x}dy^2 + C^2e^{2x}dz^2 \tag{1}$$

where A, B, C are metric potentials and function of time t-alone.

Einstein's field equation is given by

$$R_i^j - \frac{R}{2}g_i^j = -T_i^j \tag{2}$$

(in gravitational unit $8\pi G = 1$, c=1)

where T_i^j is the energy momentum tensor for a cloud of strings given by Letelier (1979, 1983) as

$$T_i^j = (p+\rho)v_iv^j + pg_i^j - \lambda x_i x^j + E_i^j \tag{3}$$

with

$$v_iv^i = -x_ix^i = -1 \text{ and } \quad v^ix_i = 0 \tag{4}$$

where λ is the string tension density, ρ is the matter density, p is the thermodynamical pressure, v^i is the four-velocity vector and x^i is the direction of string. The particle density concerned with the configuration is given by

$$\rho_p = \rho - \lambda \tag{5}$$

E_i^j is the electromagnetic field given by

$$E_i^j = g^{rs}F_{ir}F_s^j - \frac{1}{4}F_{rs}F^{rs}g_i^j \tag{6}$$

We assume that the current is flowing along x-axis so magnetic field is in the yz-plane. Thus F_{23} is the only non-vanishing component of F_{ij}.

The Maxwell's equation

$$\frac{\partial}{\partial x^j}\left(F^{23}\sqrt{-g}\right) = 0 \tag{7}$$

leads to

$$F_{23} = K \tag{8}$$

where K is constant.

Now the non-vanishing components of E_i^j corresponding to the metric (1) are given as follow :

$$-E_1^1 = E_2^2 = E_3^3 = -E_4^4 = \frac{K^2}{2B^2C^2} \tag{9}$$

In the above v^i is the flow vector satisfying

$$g_{ij}v^iv^j = -1 \tag{10}$$

and direction of string is along x-axis so that $x_1 \neq 0$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$.

We assume the coordinates to be comoving so that

$$v^1 = v^2 = v^3 = 0 \text{ and } v^4 = 1 \tag{11}$$

The Einstein's field equation (2) for the metric (1) leads to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{1}{A^2}=-p+\lambda+\frac{K^2}{2B^2C^2} \quad (12)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}-\frac{1}{A^2}=-p-\frac{K^2}{2B^2C^2} \quad (13)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}=-p-\frac{K^2}{2B^2C^2} \quad (14)$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{1}{A^2}=\rho+\frac{K^2}{2B^2C^2} \quad (15)$$

and

$$\left[\frac{B_4}{B}-\frac{C_4}{C}\right]=0 \quad (16)$$

Equation (16) leads to

$$B = lC \quad \dots (17)$$

where l is an integrating constant. We consider B=C, taking $l=1$ without loss of generality then the field equations (12) to equation (15) reduces to

$$2\frac{C_{44}}{C}+\frac{{C_4}^2}{C^2}+\frac{1}{A^2}=-p+\lambda+\frac{K^2}{2C^4} \quad (18)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}-\frac{1}{A^2}=-p-\frac{K^2}{2C^4} \quad (19)$$

$$\frac{{C_4}^2}{C^2}+2\frac{A_4C_4}{AC}-\frac{1}{A^2}=\rho+\frac{K^2}{2C^4} \quad (20)$$

The spatial volume V is given by

$$V = AC^2 \quad (21)$$

The expansion (θ) and shear scalar (σ) are given by

$$\theta=\frac{A_4}{A}+2\frac{C_4}{C} \quad (22)$$

$$\sigma=\frac{1}{\sqrt{3}}\left(\frac{A_4}{A}-\frac{C_4}{C}\right) \quad (23)$$

The directional Hubble parameters in the direction of x, y and z respectively for the Bianchi type-VI_0 metric are

$$H_x=\frac{A_4}{A},\ H_y=\frac{B_4}{B}\ \text{and}\ H_z=\frac{C_4}{C} \quad (24)$$

The generalized mean Hubble parameter H is given by

$$H=\frac{1}{3}\left(H_x+H_y+H_z\right) \quad (25)$$

Deceleration parameter q is given by

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right) \quad (26)$$

3. Solutions of the Field Equations :

Here we have three non-linear differential equations (18) - (20) in five unknown parameters A, C, p, λ and ρ. To obtain coherent solutions of the system of equations two spare stipulations are required. To get the deterministic model, we assume that the expansion (θ) is proportional to the shear (σ).

This leads to condition

$$A = C^n \tag{27}$$

And we assume the physically presumable condition that the fluid is anti-stiff fluid *i.e.*

$$p + \rho = 0 \tag{28}$$

From equation (19) and equation (20), after using equation (28), we get

$$\frac{{C_4}^2}{C^2} + \frac{A_4 C_4}{AC} - \frac{A_{44}}{A} - \frac{C_{44}}{C} = \frac{K^2}{C^4} \tag{29}$$

Substituting equation (27) into equation (29), we obtain

$$C_{44} + \frac{(n^2-2n-1)}{(n+1)}\frac{{C_4}^2}{C} = -\frac{K^2}{(n+1)C^3} \tag{30}$$

Let us consider $C_4 = f(C)$ and

$C_{44} = ff^1$, $f^1 = \frac{df}{dC}$ in equation (30)

We get,

$$ff^1 + \frac{(n^2-2n-1)}{(n+1)}\frac{f^2}{C} = -\frac{K^2}{(n+1)C^3} \tag{31}$$

Equation (31) can be written as

$$\frac{df^2}{dC} + \frac{2(n^2-2n-1)}{(n+1)}\frac{f^2}{C} = -\frac{2K^2}{(n+1)C^3} \tag{32}$$

On integrating equation (32), we get

$$f^2 = {C_4}^2 = -\frac{K^2}{C^2(n^2-3n-2)} + SC^{\frac{-2(n^2-2n-1)}{(n+1)}} \tag{33}$$

where S is the constant of integration.

Equation (33) leads to

$$\int \frac{dC}{\sqrt{SC^{\frac{-2(n^2-2n-1)}{(n+1)}} - \frac{K^2}{C^2(n^2-3n-2)}}} = \int dt + M = t + M \tag{34}$$

where M is the integrating constant.

Appropriate transformation of coordinates

C=T, x=X, y=Y, z=Z

The metric (1) becomes,

$$ds^2 = \frac{dT^2}{-\frac{K^2}{T^2(n^2-3n-2)} + S\,T^{\frac{-2(n^2-2n-1)}{(n+1)}}} + T^{2n}dX^2 + T^2e^{-2X}dY^2 + T^2e^{2X}dZ^2 \tag{35}$$

4. Special Models :

I. For n=2 and S=0 in the presence of magnetic field

To obtain the deterministic solution in terms of cosmic time 't', we put n = 2 and S = 0 in equation (33) which leads to

$$f^2 = {C_4}^2 = \frac{K^2}{4C^2} \tag{36}$$

Equation (36) can be rewrite as

$$\int 2CdC = \int Kdt \tag{37}$$

On integrating equation (37), we get

$$C = \sqrt{T} \tag{38}$$

where $Kt + N = T$ (39)

and N is the integrating constant.

Therefore, metric (1) reduces to the form

$$ds^2 = -\frac{dT^2}{K^2} + T^2dX^2 + T(e^{-2X}dY^2 + e^{2X}dZ^2) \tag{40}$$

5. The Geometrical and Physical outcomes of model in the presence of Magnetic Field :

The energy density (ρ), isotropic pressure (p), string tension density (λ), particle density (ρ_p) attached to the string, the expansion (ρ), shear (σ), Hubble directional parameters (H_x, H_y and H_z), Hubble parameter (H), deceleration parameter (q) and spatial volume (V) for the model (35) are given by

$$\rho = (2n+1)\frac{S}{T^{\frac{2n(n-1)}{(n+1)}}} - \frac{K^2(n^2+n)}{2(n^2-3n-2)\,T^4} - \frac{1}{T^{2n}} \tag{41}$$

$$p = -\left((2n+1)\frac{S}{T^{\frac{2n(n-1)}{(n+1)}}} - \frac{K^2(n^2+n)}{2(n^2-3n-2)\,T^4} - \frac{1}{T^{2n}}\right) \tag{42}$$

$$\lambda = \frac{2(-2n^2+n+1)}{(n+1)}\frac{S}{T^{\frac{2n(n-1)}{n+1}}} + \frac{2K^2(n+1)}{(n^2-3n-2)T^4} + \frac{2}{T^{2n}} \tag{43}$$

$$\rho_p = \frac{(6n^2+n-1)}{(n+1)} \frac{S}{T^{\frac{2n(n-1)}{n+1}}} - \frac{K^2(n^2+5n+4)}{2(n^2-3n-2)T^4} - \frac{3}{T^{2n}} \tag{44}$$

$$\theta = (n+2)\sqrt{\frac{S}{T^{\frac{2n(n-1)}{n+1}}} - \frac{K^2}{(n^2-3n-2)T^4}} \tag{45}$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\sqrt{\frac{S}{T^{\frac{2n(n-1)}{n+1}}} - \frac{K^2}{(n^2-3n-2)T^4}} \tag{46}$$

From (45) and (46), we get

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(n+2)} = \text{constant, } (n \neq -2) \tag{47}$$

$$H_x = n\left(\sqrt{\frac{S}{T^{\frac{2n(n-1)}{n+1}}} - \frac{K^2}{(n^2-3n-2)T^4}}\right) \tag{48}$$

$$H_y = H_z = \sqrt{\frac{S}{T^{\frac{2n(n-1)}{n+1}}} - \frac{K^2}{(n^2-3n-2)T^4}} \tag{49}$$

$$H = \frac{n+2}{3}\left(\sqrt{\frac{S}{T^{\frac{2n(n-1)}{n+1}}} - \frac{K^2}{(n^2-3n-2)T^4}}\right) \tag{50}$$

$$q = -1 + \frac{3}{n+2}\left\{\frac{\frac{2k^2}{(n^2-3n-2)T^4} - \frac{n(n-1)}{(n+1)T^{\frac{2n(n-1)}{n+1}}}S}{\frac{k^2}{(n^2-3n-2)T^4} - \frac{1}{T^{\frac{2n(n-1)}{n+1}}}S}\right\} \tag{51}$$

Energy condition $\rho \geq 0$ leads to

$$(2n+1)\frac{S}{T^{\frac{2n(n-1)}{(n+1)}}} - \frac{K^2(n^2+n)}{2(n^2-3n-2)\,T^4} - \frac{1}{T^{2n}} \geq 0 \tag{52}$$

$$V = T^{n+2} \tag{53}$$

The magnitude of rotation

$$\omega = 0 \tag{54}$$

Now, the energy density (ρ), isotropic pressure (p), string tension density (λ), particle density (ρ_p) attached to the string, the expansion (θ), shear (σ), Hubble directional parameters (H_x, H_y and H_z), Hubble parameter (H), deceleration parameter (q) and spatial volume (V) for the model (40) are given by

$$\rho = \frac{3K^2-4}{4T^2} \tag{55}$$

$$p = -\left(\frac{3K^2-4}{4T^2}\right) \tag{56}$$

$$\lambda = -\left(\frac{3K^2-4}{2T^2}\right) \tag{57}$$

$$\rho_p = \frac{3(3K^2-4)}{4T^2} \tag{58}$$

$$\theta = \frac{2K}{T} \tag{59}$$

$$\sigma = \frac{K}{2\sqrt{3}T} \tag{60}$$

From (59) and (60), we get

$$\frac{\sigma}{\theta} = \frac{1}{4\sqrt{3}} = \text{constant} \tag{61}$$

$$H_x = \frac{K}{T} \tag{62}$$

$$H_y = H_z = \frac{K}{2T} \tag{63}$$

$$H = \frac{2K}{3T} \tag{64}$$

$$q = \frac{1}{2} \tag{65}$$

$$V = T^{1+\frac{n}{2}} \tag{66}$$

The magnitude of rotation

$$\omega = 0 \tag{67}$$

Conclusion

In this paper, we have investigated Magnetized Bianchi type-VI_0 string cosmological model for anti-stiff fluid in General Relativity. By assuming anti-stiff fluid condition and the relation between metric potential, we quest the comprehensive solutions of the Einstein's field equations. The model (35) starts expanding with big bang at T=0 and the expansion of the model decreases as time increases for n > 1 and it approaches to zero as $T\to\infty$ and stops when n = -2. Since $T\to\infty$, $\frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large value of T. However, if n = 1 then $\sigma = 0$. Therefore the model isotropizes for n = 1.

For large value of T, declaration parameter q < 0 for positive integral value of n except n = 4 which show the accelerating phase of the universe. The Hubble parameter of the model also decreases as time increases. We also observed that all parameters tend to 0 as $T \to \infty$, for $n > 1$. The energy condition $\rho \geq 0$ is satisfied for all values of T. As T tends to 0, V tends to zero and V is increasing function of T for $n > -2$.

In the model (40) expansion is decreasing function of time T and stop as $T \to \infty$. All physical parameters are decreasing function of time T. The value of deceleration parameter (q) obtained is 0.5 which show the decelerating phase of the universe.

The model (35) has Point Type singularity for $n > 0$ at $T = 0$. All physical parameters decrease more rapidly in the presence of magnetic field. In general, the present model represents shearing, expanding and non-rotating universe.

Scope and application :

Magnetic field plays a crucial role in star and galaxy formation and evolution. The effect of magnetic field upon the dynamics of universe is necessary during early and late time evolution of the universe.

The present investigation will be very helpful to the researchers who are engage for the research work in cosmological models in the presence of magnetic field.

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