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Section A

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Bianchi Type-I Inflationary Cosmological Models With Constant Deceleration Parameter For Perfect Fluid Distribution In General Relativity

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Abstract

The models represent expanding and shearing universe and both models start with Big Bang initially and the expansion stops for large time. The spatial volume increases with time representing inflationary scenario. The shear is zero for large value of time representing isotropization of universe. The Hubble parameter determines the age of the universe. The expression for Hubble parameter matches with FRW model.

Key words : Bianchi Type-I, Inflationary Universe, Perfect fluid, Cosmology (85A40).

1. Introduction

Many cosmological problems are being investigated by cosmologists to understand the early evolution of universe. Einstein's theory of general relativity[12] has provided sophisticated theory of gravitation. The inflationary scenario explains several mysteries of modern cosmology like homogeneity, isotropy, horizon problem and the flatness of the observed universe. Inflation means extremely rapid expansion of early universe by a factor of 10^{78} in volume driven by negative pressure vacuum energy density. Guth's[13] introduced the concept of inflation and suggested that rapid expansion is due to false vacuum energy and after inflation, the universe is filled with bubbles. Various authors viz. Abbott and Wise[1], Abrecht and Steinhardt[2], Burd and Barow[11], La and Steinhardt[14], Linde[15], Mijic *et al.*[16] have investigated inflationary cosmological models using homogeneous and isotropic space-time in different versions. The objective of the present paper is to investigate inflationary scenario using massless scalar field and flat potential in the framework of anisotropic Bianchi type –I space time in general relativity.

Rothman and Ellis[18] explained that we could have a solution of the isotropy problem if we work with anisotropic metric and these can be inflated in a very general circumstances. Stein-Schabes[20] has pointed out that inflationary scenario is possible when potential has flat region and higgs field evolves slowly but the universe expands in an exponential way due to vacuum field energy. Bali[3,4], Bali and Jain[5], Bali and Poonia[6] have discussed inflationary cosmological models in general relativity using Bianchi Type-I space time in which the potential is considered as constant. Inflationary scenario in Bianchi Type-V space time with bulk viscosity and dark energy in radiation dominated phase investigated by Bali and Goyal[7]. Many authors viz. Bali and Tyagi[10], Bali and Saraf[8], Naidu *et al.*[17], Tripathi *et al.*[21], Tyagi and Singh[23] and Tyagi *et al.*[22] have studied various Bianchi Type models in different context.

Inspired by the above exploration, in this paper, we have investigated the Bianchi Type-I inflationary cosmological models with perfect fluid distribution in the presence of massless scalar field with a flat region in which potential is constant. The other physical aspects of the models are also discussed.

2. The metric and Field Equations :

We consider the space-time admitting Bianchi Type-I group of motions in the form as :

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \tag{1}$$

Where A, B and C are metric potential and are function of t alone.

The action of gravitational field minimally coupled to a scalar field with potential $V(\phi)$ is given by Stein-Schabes[20], we have

$$L = \int \sqrt{-g}\left[R - \frac{1}{2} g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right] d^4X \tag{2}$$

In a co-moving coordinate system, we have

$$v^i = (0, 0, 0, 1); \tag{3}$$

The Einstein's field equation (in the gravitational unit c=8πG=1) are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{4}$$

with

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} + \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_w\phi\partial^w\phi + V(\phi)\right] g_{ij} \tag{5}$$

Here ρ is proper energy density, p the isotropic presence ϕ the Higgs field, V the potential, v^i is the unit time like vector.

The conservation relation leads to

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\phi\right)=-\frac{dV}{d\phi} \tag{6}$$

The Einstein's field equations (5) for metric (1) leads to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}=-p-\frac{1}{2}\phi_4{}^2+V(\phi) \tag{7}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}=-p-\frac{1}{2}\phi_4{}^2+V(\phi) \tag{8}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=-p-\frac{1}{2}\phi_4{}^2+V(\phi) \tag{9}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}=\rho+\frac{1}{2}\phi_4{}^2+V(\phi) \tag{10}$$

The equation (6) for scalar field (ϕ) leads to

$$\phi_{44}+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\phi_4=0 \tag{11}$$

where suffix '4' indicates ordinary partial derivative with respect to t.

3. Solutions of Field Equations :

The field equation (7) - (10) are system of four equation's with unknown parameters A, B, C, ρ, p, ϕ. To obtain the deterministic solution, we assume the following conditions :

(i) $V(\phi)$ is constant

i.e. $V(\phi) = K$ (12)

(ii) Shear (σ) is proportional to expansion (θ) which leads to

$A = (BC)^n$ (13)

Equation (11) and (13) we have

$$\phi_4=\frac{m_1}{A^{\frac{n+1}{n}}} \tag{14}$$

where m_1 is constant of integration.

The scale factor R^3 for line-element (1) is given by

$$R^3=A^{\frac{n+1}{n}} \tag{15}$$

To find the deterministic model of the universe, we assume that

q= Deceleration parameter = Constant = α

Thus we have

$$q = -\frac{\frac{R_{44}}{R}}{\frac{R_4^2}{R^2}} = \alpha \text{ (Constant)} \tag{16}$$

Case (i): $\alpha \succ 0$ i.e. $\alpha = a$ then we have

$$A = (\beta t + \gamma)^{\frac{1}{b+1}} \tag{17}$$

where $b = [\frac{(1-2n)+a(n-1)}{3n}]$, $\beta = (b+1)m_2$, $\gamma = (b+1)m_3$

and m_2, m_3 are constant of integration.

Using condition (12) and (13) in field equation (8) and (9), we get

$$C^2\left(\frac{B}{C}\right)_4 = \frac{m_4}{A} \tag{18}$$

Where m_4 is constant of integration.

Now we assume that $BC = \mu$ and $\frac{B}{C} = \nu$, then we get

$$\nu = m_5 e^{h(\beta t+\gamma)^{\frac{nb-1}{n(b+1)}}} \tag{19}$$

Where $h = \frac{nm_4}{m_2(nb-1)}$ and m_5 is constant of integration.

Hence, by suitable transformation of coordinates, metric (1) reduces to

$$ds^2 = -\frac{1}{\beta^2}dT^2 + T^{\frac{1}{n(b+1)}}\left[e^{bT^{\frac{nb-1}{n(b+1)}}}dY^2 + e^{hT^{\frac{nb-1}{n(b+1)}}}dZ^2\right] \tag{20}$$

Where x=X, $\sqrt{m_5}\,y = Y$, $\frac{z}{\sqrt{m_5}} = Z$ and $(\beta t+\gamma) = T$

Case (ii): $\alpha \prec 0$ i.e. $\alpha = -a$ then we have

$$A = (\lambda t + \delta)^{\frac{1}{1-S}} \tag{21}$$

where $S = \left[\frac{a(n+1)-(1-2n)}{3n}\right]$, $\lambda = m_6(1-S)$, $\delta = m_7(1-S)$ and m_6, m_7 are constant of integration.

and

$$\nu = m_8 e^{-W(\lambda t+\delta)^{\frac{nS+1}{n(S-1)}}} \tag{22}$$

where $W = \dfrac{nm_4}{m_6(nS+1)}$ and m_8 is constant of integration.

Hence, by suitable transformation of coordinates, metric (1) reduces to

$$ds^2 = -\frac{1}{\lambda^2}d\tau^2 + \tau^{\frac{2}{1-S}}dX^2 + \tau^{\frac{1}{n(1-S)}}\left[e^{-W\tau^{\frac{nS+1}{n(S-1)}}}dY^2 + e^{W\tau^{\frac{nS+1}{n(S-1)}}}dZ^2\right] \tag{23}$$

Where $x = X$, $\sqrt{m_8}\,y = Y$, $\dfrac{z}{\sqrt{m_8}} = Z$ and $(\lambda t+\delta) = \tau$

5. Physical and Geometrical Aspects :

For the model (20), the rate of Higgs field

$$\phi = \frac{m_1 n}{m_2(nb-1)}T^{\frac{nb-1}{n(b+1)}} + m_9 \tag{24}$$

where m_9 is constant of integration.

The spatial volume

$$R^3 = T^{\frac{n+1}{n(b+1)}} \tag{25}$$

The expansion

$$\theta = (n+1)\frac{m_2}{nT} \tag{26}$$

The shear

$$\sigma = \frac{1}{\sqrt{3}}\sqrt{\left[\left(n^2 - n + \frac{1}{4}\right)\frac{m_2^2}{n^2T^2} + \frac{3m_4^2}{4}T^{\frac{-2(n+1)}{n(b+1)}}\right]} \tag{27}$$

The Hubble parameter

$$H = \frac{m_2(n+1)}{3nT} \tag{28}$$

and

$$\frac{\sigma}{\theta} = \frac{n}{\sqrt{3}m_2(n+1)}\sqrt{\left[\left(n^2 - n + \frac{1}{4}\right)\frac{m_2^2}{n^2} + \frac{3m_4^2}{4}T^{\frac{2(nb-1)}{n(b+1)}}\right]} \tag{29}$$

Energy density

$$\rho = \frac{m_1^2(4n+1)}{4n^2T^2} - \frac{(m_3^2 + 2m_1^2)}{4T^{\frac{2(n+1)}{n(b+1)}}} - K \tag{30}$$

And pressure

$$p = K + \frac{(2m_1^2 - m_3^2)}{4T^{\frac{2(n+1)}{n(b+1)}}} - \frac{m_2 m_4 (b+1)}{n}\log T - \frac{3m_1^2}{4n^2T^2} \tag{31}$$

For the model (23), the rate of Higgs field

$$\phi = -\frac{m_1 n}{m_6(nS+1)}\tau^{\frac{nS+1}{n(S-1)}} + m_{10} \tag{32}$$

where m_{10} is integrating constant.

The spatial volume

$$R^3 = \tau^{\frac{n+1}{n(1-S)}} \tag{33}$$

The expansion (θ)

$$\theta = (n+1)\frac{m_6}{n\tau} \tag{34}$$

The shear

$$\sigma = \frac{1}{\sqrt{3}}\sqrt{\left[\left(n^2 - n + \frac{1}{4}\right)\frac{m_6^2}{n^2\tau^2} + \frac{3m_4^2}{4}\tau^{-\frac{2(n+1)}{n(1-S)}}\right]} \tag{35}$$

And

$$\frac{\sigma}{\theta} = \frac{n}{\sqrt{3}m_6(n+1)}\sqrt{\left[\left(n^2 - n + \frac{1}{4}\right)\frac{m_6^2}{n^2} + \frac{3m_4^2}{4}\tau^{\frac{2(nS+1)}{n(S-1)}}\right]} \tag{36}$$

The Hubble parameter

$$H = \frac{(n+1)m_6}{3n\tau} \tag{37}$$

Energy density

$$\rho = \frac{(4n+1)m_6^2}{4n^2\tau^2} - \frac{(m_4^2 + 2m_1^2)}{4\tau^{\frac{2(n+1)}{n(1-S)}}} - K \tag{38}$$

Isotropic pressure

$$p = K + \frac{m_1^2}{2\tau^{\frac{2(n+1)}{n(1-S)}}} - \frac{m_6^2(1-S)}{n}\log\tau - \frac{m_4^2}{4\tau^{\frac{2(n+1)}{n(1-S)}}} - \frac{3m_6^2}{4n^2\tau^2} \quad \ldots \quad (39)$$

6. Discussion

The Spatial Volume (R^3) for the both models (20) and (23), increases with time (when $T \to \infty, R^3 \to \infty$ and $\tau \to \infty$, $R^3 \to \infty$). It represents inflationary scenario of universe containing massless scalar field with flat potential and constant deceleration parameter. Both models start with Big Bang at $T = 0$ and expansion stops ultimately ($T \to \infty$). Since $\frac{\sigma}{\theta} \to$ finite term for large value of time, it shows that the both models are approaches anisotropy. The Hubble parameter decreases with time in both models. The energy density and pressure of both models are initially large but decreases with time.

7. Conclusion

We have investigated Bianchi Type-I Inflationary universe under taken in framework of massless scalar field with flat potential and constant deceleration parameter in presence of perfect fluid distribution. To get the deterministic solution of the model, we assumed that (i) The potential V(ϕ) is constant and (ii) $A = (BC)^n$. The models lead to eternal inflation, in presence of perfect fluid. Some geometrical and Physical aspects of the models are also discussed in detail. The present investigation will be very helpful to the researchers who are engage for the research work in inflationary cosmology.

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