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# Mathematical Modeling of Impact of Pollutants on Industries Based on Resource Biomass

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**Abstract**

Depletion of resources such as forestry, minerals etc. and resource-based industries such as wood and paper etc., due to rising pollution, is one of the biggest challenges which the humankind is facing today. In this paper, a mathematical model has been designed to give an insight into the effect of pollutants on natural resources which in turn affects the growth and stability of industries dependent on such biomass. The model is analyzed using stability theory of differential equations. Five dependent variables are considered in the model and some important assumptions are made. Two equilibria are found in the equilibrium analysis and conditions of local and global stability of interior equilibrium are obtained. Numerical simulation is also done to demonstrate the analytical findings. It is found in the study that as we impose an environmental tax on the polluters, the concentration of pollutants in the environment is controlled and the stable equilibrium shifts in such a way that the densities of resource biomass and dependent industries are close to the densities which correspond to the pollution free ecosystem.

*Key words :* Pollutants, Resource biomass, Industry, Mathematical model, Stability analysis.
2020 Mathematics Subject Classification No.: **92D40**

## Introduction

The major cause of concern for the society seems to be the increasing concentration of toxic elements in the atmosphere through natural as well as human activities which disturb the ecological balance. This causes the depletion of renewable resources like timber, agricultural products, landfill gas, etc., which is posing a great threat to industries like wood and paper. In this paper, it has been considered that the industry under consideration is either partially or wholly dependent on resource biomass. Therefore, these industries get affected due to the pollutants indirectly. For instance, the fisheries and fish-based industries, where the pollutants are emitted from industries situated in coastal regions affecting the regeneration and the health of fishes and therefore pollutants not only affect the resources but also affect the emitter itself (the industry).

The pollutants enter into the atmosphere through various external sources and as their concentration increases, the densities of resource biomass and resource-dependent industries decreases. This increase in concentration of atmospheric pollutants can be curbed by imposing an environmental tax, without which, the resource biomass will become extinct in the near future.

Li *et al.*[16] have shown that the technological innovation of resource-based industries can be promoted by environmental regulation. Mearns *et al.*[1] have discussed about detrimental impact of pollutants on marine organisms in their review. Various other researchers[4-6,15] have investigated the phenomena of survival of species in polluted environment using mathematical modelling. Shukla *et al.*[8-13] have proposed various models to study the detrimental effects of industrialization on forestry resources. They have modeled the survival of resource dependent population affected by toxicants (pollutants) emitted from external sources as well as formed by its precursor. Rai and Malviya[7] have studied the survival of resource dependent industries when its resource is being reduced by pollutants. Malviya[2-3] have suggested models to study the survival of population which is directly affected by pollutants from various polluters. But these researchers did not consider the effect of environmental tax on resource-based industry.

In view of these, we consider five dependent variables in the modelling process. D(t) is the density of industries based on resource biomass of density B(t). T(t) and U(t) are the concentrations of the pollutant under consideration in the environment and in the uptake phase of the resource biomass respectively at any moment t. It has been assumed that the concentration of the pollutant from the different sources is dependent on the tax imposed on the emitters. This factor has been considered as $[Q - \rho . I]$ where Q and $\rho$ are constants and $\mathrm{I(t)}$ is the environment tax, introduced to control the emission of pollutants. $\beta_1$ is the growth rate coefficient of the industries based on the resource-biomass. q and $q_0$ are the factors which regulate the growth of the industries in the ecosystem under consideration. It has been assumed that the resource biomass is growing intrinsically with the rate $s$, which is being affected by the concentration of the pollutants in the resource biomass. Thus, $s$ is function of U which is the uptake concentration of the pollutant in resource biomass. Since the intrinsic growth of the resource biomass decreases with the presence of the pollutants,

$$\frac{ds(U)}{dt} < 0 \, for U \geq 0 \tag{1.1}$$

Let $s(0) = s_0 > 0$, then it is clear that the $s_0$ will be maximum of $s(U)$. $\beta_2$. is depletion rate coefficient for the resource biomass. The toxicant is also depleting in the environment and in the resource biomass due to various internal physiological phenomenon, we are assuming the depletion rate coefficients of the pollutant in the environment as $\delta$ and in the resource biomass as $\phi$. $\alpha$ is considered as depletion rate of the pollutants in the environment due to uptake of the pollutants by the resource biomass under consideration. Let the vanishing rate of resource biomass due to pollutants be $\nu$ and a fraction $\pi$ of this be turning back into in the environment. Let $\alpha$BT be the rate with which concentration of the pollutant in the environment is decreasing. Also assume that k is fraction of $\alpha BT$ with which the resource biomass is being affected then k$\alpha$BT will be rate of decrease of B. $\nu UB$ is the rate with which concentration of the pollutants in the resource biomass is depleting and $\pi\nu UB$ is the rate with which the pollutants are turning back in the environment.

Let the function L(T) denote the maximum density of the resource biomass, then with the increase of concentration of pollutants in the environment, this function decreases *i.e.*

$$\frac{dL(T)}{dT} < 0 \, for T \geq 0 \tag{1.2}$$

Suppose that $L(0) = L_0 > 0$, then it is obvious that the $L_0$ will be maximum of L(T). We have assumed that if at a particular time the concentration of the pollutants in the ecosystem (T) is less than the a certain permissible limit $T_0$, then no tax would be charged from the various industries. θ and $\theta_0$ are the constants, where $\theta_0 I$ is the factor which has been considered due to some practical difficulties on implementing the foolproof tax system.

In this analysis following assumptions have been made:

1. The growth of industries based on biological resources and density of resource biomass are assumed to be governed by logistic equations with respective intrinsic growth rates.
2. The rate of emission of pollutants or toxicants may fluctuate depending upon the working of industries and various activities of the human population. But here, it is assumed that the cumulative rate of the introduction of the pollutants from the external sources is constant when no tax is charged from emitters but as the taxation system is introduced, it depends on the total amount of taxes collected.
3. The concentration of the pollutants or toxicants in the environment decreases due to their integration, absorption, deposition, etc. by resource biomass. This amount being proportional to resource biomass density and the environmental concentration of the toxicant.
4. A fraction of the total amount of the pollutants or toxicants becomes part of the uptake phase in the resource biomass.
5. The environmental concentration of the pollutants reduces the carrying capacity of resource biomass.

**2. Mathematical Model :**

In view of the above-mentioned assumptions and variables considered in the modeling process, we propose the following non-linear mathematical model-

$$\begin{aligned}
\frac{dD}{dt} &= q - q_0 D + \beta_1 DB \\
\frac{dB}{dt} &= s(U)B - \frac{s_0 B^2}{L(T)} - \beta_2 DB - k\alpha BT \\
\frac{dT}{dt} &= Q - \rho I - \delta T - \alpha BT + \pi \nu UB \\
\frac{dU}{dt} &= (1-k)\alpha BT - \phi U - \nu UB \\
\frac{dI}{dt} &= \theta(T - T_0) - \theta_0 I
\end{aligned} \tag{2.1}$$

$$\frac{dr(U)}{dU} < 0 \, for U \geq 0 \tag{2.2a}$$

$$\frac{dK(T)}{dT} < 0\, for T \geq 0 \tag{2.2b}$$

$$Where D(0) = D_0 \geq 0; B(0) = B_0; T(0) = T_0 \geq 0; U(0) = U_0 \geq 0; s(0) = s_0; L(0) = L_0;$$
$$q; q_0; \alpha; \beta_1; \beta_2; Q; \delta; \nu; \rho; \varphi; \theta; \theta_0; T_0 > 0; 0 \leq k; \pi \leq 1$$

*3. Equilibrium Analysis*

There are two equilibrium points $E_0$ and $E_1$ in this system. These are as follows:

$$(1.) E_0(D,0,T,0,I) = E_0[\frac{q}{q_0}, 0, \frac{Q\theta_0 + \rho\theta T_0}{\delta\theta_0 + \rho\theta}, 0, \frac{Q\theta - \delta\theta T_0}{\rho\theta + \delta\theta_0}]$$
$$(2.) E_1(D^*, B^*, T^*, U^*, I^*)$$

(where, $\delta\theta_0 + \rho\theta \neq 0$ and $q_0 \neq 0$ since all these parameters are positive and >0).

The existence of $E_0$ is obvious and existence of $E_1$ is as follows :

Equating to zero the right-hand side expressions of model (2.1), we get

$$D = \frac{q}{q_0 - \beta_1 B} = f(B) \text{ (say)} \tag{3.1a}$$

$$B = \frac{s(U)L(T) - \beta_2 DL(T) - k\alpha TL(T)}{s_0} \tag{3.1b}$$

From the second equation of the system (2.1), $\frac{dB}{dt}$ can be written as:

$$\frac{dB}{dt} = (s(U) - \beta_2 D - k\alpha T)B - \frac{s_0 B^2}{L(T)}$$

Thus $(s(U)-\beta_2 D-k\alpha T)$ denotes the intrinsic growth rate of $\frac{dB}{dt}$ and this will be an increasing function only if $(s(U)-\beta_2 D-k\alpha T)>0$, where $D, U, T \geq 0$.

$$T = \frac{(Q\theta_0 + \rho\theta T_0)(\phi + \nu B)}{f_1(B)} = g(B) \text{ (say)} \tag{3.1c}$$

$$U = \frac{(Q\theta_0 + \rho\theta T_0)(1-k)\alpha B}{f_1(B)} = h(B) \text{ (say)} \tag{3.1d}$$

$$I = \frac{\theta}{\theta_0}[g(B) - T_0] = i(B) \text{ (say), provided } g(B) > T_0 \tag{3.1e}$$

Here $f_1(B) = \phi(\delta\theta_0 + \rho\theta) + [\phi\alpha\theta_0 + \nu(\delta\theta_0 + \rho\theta)]B + \alpha\nu[\theta_0 - (1-k)\pi]B^2$

Let F(B) be such that:

$$F(B) = s_0 B - s(h(B)L(g(B)) + \beta_2 f(B)L(g(B)) + k\alpha g(B)L(g(B)) \quad (3.1f)$$

From the equations (3.1a) to (3.1e) and putting B=0 and $L_0$ in (3.1f), we get:

$$F(0) = -L\left(\frac{Q\theta_0 + \rho\theta T_0}{\delta\theta_0 + \rho\theta}\right)\left[s_0 - \beta_2 \frac{q}{q_0} - k\alpha\left(\frac{Q\theta_0 + \rho\theta T_0}{\delta\theta_0 + \rho\theta}\right)\right]$$

i.e. $F(0) = -L(T)[s_0 - \beta_2 D - k\alpha T]$ at B=0

Thus $F(0) < 0$; since $L(T)$ being carrying capacity of resource biomass is always positive and $(s(U)-\beta_2 D-k\alpha T) > 0$ as concluded earlier.
Now, we find the sign of $F(L_0)$;
In this case $L_0$ has been taken at $T = 0$, therefore, $g(L_0) = 0$,

$$F(L_0) = L_0[s_0 - s(h(L_0)) + \beta_2 f(L_0)]$$

Here, $s_0$ has been taken as maximum of $s$, therefore first two terms give a positive value and $\beta_2$ being depletion rate coefficient is always positive, thus we have, $F(L_0) > 0$;
It can be concluded that there exists a root $B^*$ of the equation (3.1f) in the interval $0 < B < L_0$.
To show the uniqueness, we find $F'(B)$ as follows-

$$F(B) = s_0 B - s(h(B))L(g(B)) + \beta_2 f(B)L(g(B)) + k\alpha g(B)L(g(B))$$

$$F'(B) = s_0 + \frac{dL}{dg}\frac{dg}{dB}[-s(h(B) + \beta_2 f(B) + k\alpha g(B)] + L(g(B))\left[-\frac{ds}{dh}\frac{dh}{dB} + \beta_2[\frac{q\beta_1}{(q_0 - \beta_1 B)^2}] + k\alpha\frac{dg}{dB}\right]$$

Now since $F(B) = 0$ at $B = B^*$, we have from (3.1f),

$$F'(B) = s_0 + \left[-s_0\frac{B}{L(g(B))}\right]\frac{dL}{dg}\frac{dg}{dB} - L(g(B))\frac{ds}{dh}\frac{dh}{dg} + k\alpha L(g(B))\frac{dg}{dB} + \beta_2\left[\frac{q\beta_1}{(q_0 - \beta_1 B)^2}\right]L(g(B))$$

This should be > 0 for the root $B$ to be unique. When $B^*$ is determined, $D^*$, $T^*$, $U^*$ and $I^*$ can also be found by solving the system (3.1a) to (3.1e).

Thus, the existence of the equilibrium at $E_1(D^*, B^*, T^*, U^*, I^*)$ and the condition of its uniqueness is proved.

Now let us examine the effect of Q, as is the emission before the introduction of the taxes, i.e. the cumulative rate of introduction of the pollutants on the density of the resource biomass.
From equation (3.1b)

$$B = \frac{s(U)L(T) - \beta_2 DL(T) - k\alpha TL(T)}{s_0}$$

$$s_0 B = s(h)L(g) - \frac{\beta_2 q}{q_0 - \beta_1 B}L(g) - k\alpha g L(g)$$

Therefore

$$s_0 \frac{dB}{dQ} = s(h)\frac{dL}{dg}\frac{dg}{dQ} - \frac{\beta_2 q}{q_0 - \beta_1 B}\frac{dL}{dg}\frac{dg}{dQ} - k\alpha g\frac{dL}{dg}\frac{dg}{dQ} + L(g)\frac{ds}{dh}\frac{dh}{dQ} - \frac{\beta_1\beta_2 q}{(q_0 - \beta_1 B)^2}L(g)\frac{dB}{dQ} - k\alpha L(g)\frac{dg}{dQ}$$

Now, $\frac{dg}{dQ} = \frac{\partial g}{\partial B}\left(\frac{dB}{dQ}\right) + \frac{\partial g}{\partial Q}$

And $\frac{dh}{dQ} = \frac{\partial h}{\partial B}\left(\frac{dB}{dQ}\right) + \frac{\partial h}{\partial Q}$

Therefore, on substituting and solving we get:

$$\frac{dB}{dQ}\left[s_0 - s_0\frac{B}{L(g)}\frac{dL}{dg}\frac{\partial g}{\partial B} + L(g)\frac{\beta_1\beta_2 q}{(q_0 - \beta_1 B)^2} + k\alpha L\frac{\partial g}{\partial B} - L\frac{ds}{dh}\frac{\partial h}{\partial B}\right] = s_0\frac{B}{L(g)}\frac{dL}{dg}\frac{\partial g}{\partial Q} - k\alpha L\frac{\partial g}{\partial Q} + L(g)\frac{ds}{dh}\frac{\partial h}{\partial Q}$$

In the condition of uniqueness, we have shown that $F'(B) > 0$ in the interval $0 < B < L_0$. Further in view of equation (3.1c) and (3.1d),

We have, $\frac{\partial g}{\partial Q} = \frac{\theta_0(\phi + vB)}{f_1(B)}$ and $\frac{\partial h}{\partial Q} = \frac{\theta_0(1-k)\alpha B}{f_1(B)}$

And from equations (1.1) and (1.2),

$\frac{ds(U)}{dU} < 0 \, for U \geq 0$ and $\frac{dL(T)}{dT} < 0 \, for T \geq 0$.

From these conditions and results it is clear that:

$\frac{dB}{dQ} < 0$.

From this it is concluded that the resource biomass density decreases with the increase in the cumulative emission rate of the pollutants in the environment.

Similarly, we can show from equations (3.1a) and (3.1e), $\frac{dB}{dI} > 0$ and $\frac{dD}{dI} > 0$.

Thus, it is found that with the increase in the environment tax, the resource biomass density as well as the density of the industries dependent on the resource biomass increases.

*4. Stability Analysis* :

*Local Stability Analysis-*

For the system of equations (2.1), we find the following Jacobian matrices $M_0$ and $M_1$ at equilibrium point $E_0$ and $E_1(D^*, B^*, T^*, U^*, I^*)$ respectively.

$$M_0 = \begin{bmatrix} -q_0 & \beta_1 D & 0 & 0 & 0 \\ 0 & s_0 - \beta_2 D - k\alpha T & 0 & 0 & 0 \\ 0 & -\alpha T & -\delta & 0 & -\rho \\ 0 & (1-k)\alpha T & 0 & -\phi & 0 \\ 0 & 0 & \theta & 0 & -\theta_0 \end{bmatrix},$$

$$M_1=\begin{bmatrix} -q_0+\beta_1 B^* & \beta_1 D^* & 0 & 0 & 0 \\ -\beta_2 B^* & -\frac{s_0 B^*}{L(T^*)} & \frac{s_0 {B^*}^2 L'(T^*)}{[L(T^*)]^2}-k\alpha B^* & s'(U^*)B^* & 0 \\ 0 & -\alpha T^*+\pi\nu U^* & -\delta-\alpha B^* & \pi\nu B^* & -\rho \\ 0 & (1-k)\alpha T^*-\nu U^* & (1-k)\alpha B^* & -\phi-\nu B^* & 0 \\ 0 & 0 & \theta & 0 & -\theta_0 \end{bmatrix}.$$

From the matrix $M_0$, it is clear that $E_0(D,0,T,0,I)$ is a saddle point having stable manifold in D-U direction and unstable manifold along B- direction.

The stability behavior of $E_1(D^*,B^*,T^*,U^*,I^*)$ is not obvious from Matrix. So, we find the sufficient conditions for $E_1(D^*,B^*,T^*,U^*,I^*)$ to be locally asymptotically stable using Lyapunaov function1[4] in the following theorem -

*Theorem 4.1 :* Let the following inequality hold

$$\left[\frac{\beta_1}{\beta_2}\left(\frac{s_0 B^*}{L(T^*)^2}L'(T^*)-k\alpha\right)-C_3(\alpha T^*-\pi\nu U^*)\right]^2<2\frac{C_3\beta_1 s_0\delta}{\beta_2[L(T^*)]}$$

Where, $C_3=\dfrac{-\beta_1 s'(U^*)(1-k)\alpha}{\pi\nu\beta_2[(1-k)\alpha T^*-\nu U^*]}$ (her $\nu\neq 0$ since it is $>0$)

Then $E_1(D^*,B^*,T^*,U^*,I^*)$ will be locally asymptotically stable.

*Proof :* We consider the following positive definite function around $E_1$,

$$V=\frac{1}{2}C_1 d^2+\frac{1}{2}\frac{C_2 b^2}{B^*}+\frac{1}{2}C_3\tau^2+\frac{1}{2}C_4 u^2+\frac{1}{2}C_5 i^2$$

where $D=D^*+d, B=B^*+b, T=T^*+\tau, U=U^*+u, I=I^*+i$

It can be shown that under the inequalities mentioned in ***Theorem 4.1,*** the derivative of $V$ w .r .t. $t$ becomes negative definite. Thus $E_1$ becomes locally asymptotically stable.

*Region of attraction :*

Now we state the region of attraction of the solutions of the ecological model under consideration in the following lemma.

*Lemma 4.1 :* The following set attracts all solutions of the system (2.1) initiating in the interior of the positive octant.

$$\Gamma = \left\{ (D,B,T,U,I) : 0 \le D \le \frac{q}{q_0 - \beta_1 L_0}; 0 \le B \le L_0; 0 \le T + U \le \frac{Q}{\delta_1}; 0 \le T + U + I \le \frac{Q(1+\frac{\theta}{\delta_1})}{\delta_2} \right\}$$

$$where \delta_1 = \min\{\delta, \varphi\}; \delta_2 = \min\{\delta, \rho + \theta_0, \varphi\}$$

*Global Stability Analysis :*

*Theorem 4.2* :

Let the following conditions hold:

$L_m \le L(T) \le L_0; 0 \le -s'(U) \le q; 0 \le -L'(T) \le p$ where $L_m$, $p$ and $q$ are the positive constants in the region Γ. Then if the following two inequalities hold-

$$\left[ \frac{\beta_1 D^*}{\beta_2} \left( s_0 B^* \frac{p}{L_m{}^2} + k\alpha \right) + C_3 (\alpha T^* - \pi \nu U^*) \right]^2 < \frac{\beta_1 D^*}{\beta_2} \frac{C_3 s_0 \delta}{L_0}$$

and

$$\left[ \frac{\beta_1 D^*}{\beta_2} q + (1-k)\alpha T^* - \nu U^* \right]^2 < \frac{\beta_1 D^*}{\beta_2} \frac{s_0 \phi}{L_0}$$

where

$$C_3 = \frac{(1-k)\alpha}{\pi \nu} \qquad \text{(where } \nu \neq 0 \text{ since the same is} > 0)$$

Then equilibrium point $E_1(D^*, B^*, T^*, U^*, I^*)$ will be nonlinear asymptotically stable in the region Γ.

*Proof :* We consider the positive definite function around $E_1$

$$W(D,B,T,U,I) = \frac{1}{2} C_1 (D - D^*)^2 + C_2 \left( B - B^* - B^* \ln \frac{B}{B^*} \right) + \frac{1}{2} C_3 (T - T^*)^2 + \frac{1}{2} C_4 (U - U^*)^2 + \frac{1}{2} C_5 (I - I^*)^2$$

Differentiating w. r. t. $t$ and using model (2.1), we get

$$\dot{W} = -C_1 (q_0 - \beta_1 B)(D - D^*)^2 - \frac{C_2 s_0}{L(T)} (B - B^*)^2 - C_3 (\delta + \alpha B)(T - T^*)^2 - C_4 (\varphi + \nu B)(U - U^*)^2 - C_5 \theta_0 (I - I^*) + (C_1 \beta_1 D^* - C_2 \beta_2)(D - D^*)(B - B^*) + [C_2 \eta(U) + C_4 \{(1-k)\alpha T^* - \nu U^*\}](B - B^*)(U - U^*) + [-C_2 \{k\alpha + s_0 B^* \xi(T)\} + C_3 (\pi \nu U^* - \alpha T^*)](B - B^*)(T - T^*) + [C_3 \pi \nu B + C_4 (1-k)\alpha B](T - T^*)(U - U^*) + [-C_3 \rho + C_5 \theta](T - T^*)(I - I^*)$$

where, $\eta(U)=\begin{bmatrix}\frac{s(U)-s(U^*)}{(U-U^*)} For U \neq U^* \\ s'(U^*) For U=U^*\end{bmatrix}$ and $\xi(T)=\begin{bmatrix}\frac{\frac{1}{L(T)}-\frac{1}{L(T^*)}}{(T-T^*)} For T \neq T^* \\ -\frac{L'(T^*)}{[L(T^*)]^2} For T=T^*\end{bmatrix}$

Now taking the following assumptions into account:

$L_m \leq L(T) \leq L_0; 0 \leq -s'(U) \leq q; 0 \leq -L'(T) \leq p$ where $L_m$, $p$ and $q$ are the positive constants in the region Γ, we have:

$|\xi(T)| \leq \frac{p}{L_m^{\;2}}$ and $|\eta(U)| \leq q$

Now choosing the suitable values of constants $C_1=1$, $C_2=\frac{\beta_1 D^*}{\beta_2}$, $C_3=\frac{(1-k)\alpha}{\pi\nu}$, $C_4=1$, $C_5=\frac{C_3\rho}{\theta}$ and rearranging the terms we can show that time derivative of $W$ is negative definite function under the conditions stated in **theorem 4.2.**

*5. Numerical Simulation* :

Let us take $s(U)=s_0-\frac{a_1U(t)}{1+s_1U(t)}$, $L(T)=L_0-\frac{b_1T(t)}{1+m_1T(t)}$ and choose the following values the parameters-

$q=50, q_0=5, \beta_1=0.2, \alpha=0.01, \delta=12, k=.5, \beta_2=.1, Q=11, \pi=0.03, \nu=0.03, \phi=14,$
$\rho=5, T_0=0.5, \theta=100, \theta_0=0.005, s_0=16, a_1=1, s_1=3.9, L_0=5.62, b_1=1, m_1=1.03.$

We get the following two sets of values of variables representing

$E_0$ {D = 10, B = 0, T = 0.5000499940, U = 0, I = 0.9998800144} and

$E_1$ {D = 12.42425062, B = 4.878062053, T = 0.5000497501, U = 0.0008621570632, I = 0.9950028093}.

We have plotted the following graphs

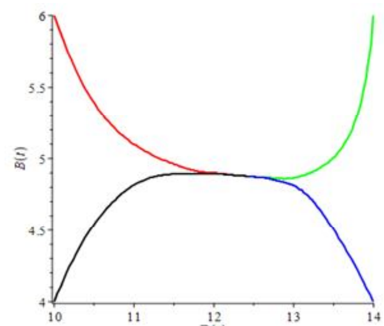


Figure 1- Density of resource biomass w. r. t. density of industries

In figure 1, it is seen that four curves with different initial conditions converge to equilibrium $E_1$. Thus, we start from any point inside the region of attraction, the solution tends to equilibrium $E_1$.

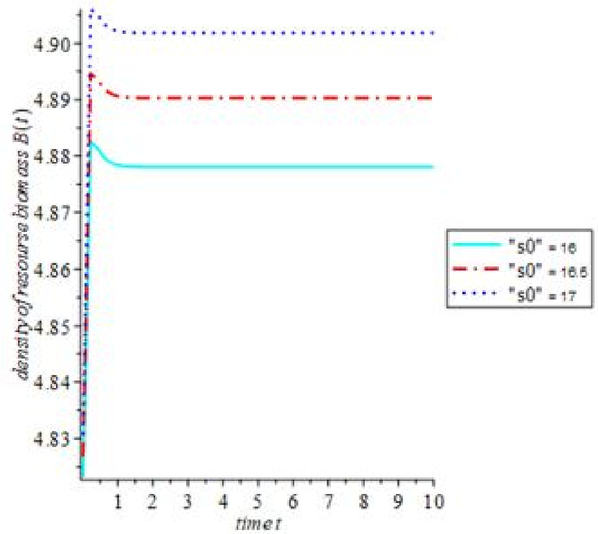


Figure 2- Density of resource biomass B(t) with respect to t taking different values of $s_0$.

In figure 2, we observe the effect of $s_0$ on density of resource biomass. It is seen that $B(t)$ increases with an increase in $s_0$.

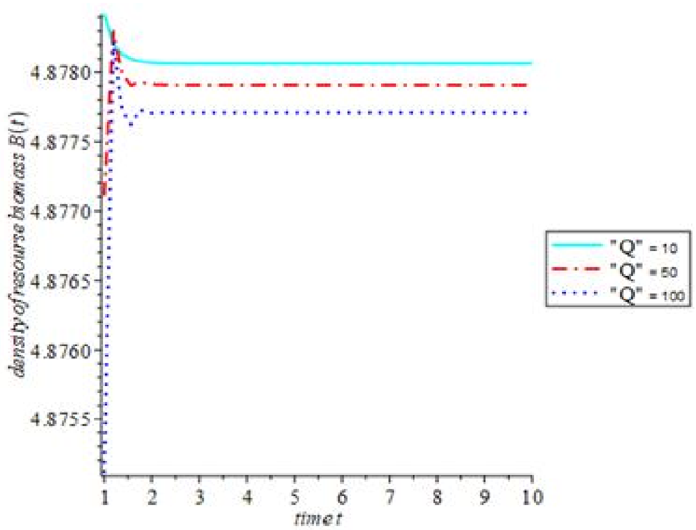


Figure 3- Density of resource biomass B(t) with respect to t taking different values of $Q$.

In figures 3, the effect of variation in density of pollutants emitted from external sources on biomass density is observed. It is seen from the figure that as Q increases, the density of resource biomass decreases. Thus, Q has a negative impact on B(t).

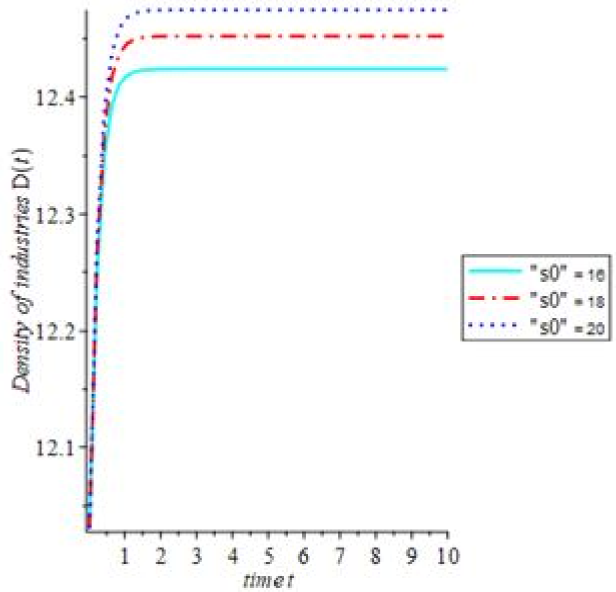


Figure 4- Density of industries D(t) with respect to t taking different values of $s_0$.

In figure 4, we observe the effect of increase in maximum intrinsic growth rate of resource biomass ($s_0$) on the density of industries D(t). It is evident that with an increase in $s_0$, the density of industries also increases.

## 5. Conclusions

It is shown through a mathematical model that on imposing an environmental tax on the polluters, the resource biomass density as well as the density of the industries increases and it may prove as an important tool for conserving resources and based industries. With the introduction of these taxes, it is observed that the densities of resources and industries are very close to the values which exist when the ecosystem is pollution free. The conclusions drawn here show that emissions of various types of pollutants in the environment must be regulated immediately through various means, otherwise resource biomass and industries reliant on resource biomass would go extinct.

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