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# Flow of Visco Elastic {Oldroyd (1958) Model} Liquid Through Porous Medium and the Annular Space Between two Right Circular Cylinders

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**Abstract**

Present paper is concerned with the oscillatory motion of visco-elastic [Oldroyd[6] model] liquid through porous medium and the annular space between two right circular cylinders perpendicular to the flow of liquid when both the cylinders execute simple harmonic motion along the central axis of the cylinders. The amplitudes and frequencies have been taken different for both cylinders. The particular cases have also been discussed in detail. The velocity of visco-elastic liquid is effected by porous medium.

*Key words* : porous medium, oscillatory, harmonically, unsteady.

**Mathematical Subject Classification-76A10**

## Introduction

Teipel[12] studied the problem of the impulsive motion of a flat plate in a visco-elastic fluid. Choubey[3] discussed the hydromagnetic flow of an electrically conducting visco-elastic Rivlin and Ericksen[11] type liquid near an infinite horizontal flat plate started impulsively from rest in its own plane with constant velocity subjected to an applied uniform transverse magnetic field. Yadav & Singh[15] studied the impulsive motion of a porous flat plate in an elastico-viscous (Rivlin-Ericksen) liquid in the presence of an uniform transverse magnetic field. The hydromagnetic flow of two immiscible visco-elastic Walter liquids between two inclined parallel plates has been studied by Chakraborty & Sengupta[2]. Krishna, Rao and Sulochana[5] have discussed the hydromagnetic oscillatory flow of a second order Rivlin Ericksen fluid in channel. Rahman and Alam Sarkar[9] studied the unsteady MHD flow of visco-elastic Oldroyd[6] fluid under time varying body force through a rectangular channel. Krishna and Rao[4] investigated magneto hydrodynamic unsteady flow through a rectangular duct with a prescribed discharge. Radhakrishnamacharya[8] studied the flow of a magnetic fluid through a non-uniform wavy tube. Arya, Kumar and Singh[1] have discussed the unsteady flow of fluid in a long uniform rectangular channel. Venkateswarlu and Narayana[14] studied MHD viscoelastic fluid over a continuously moving vertical surface with chemical reaction. Tripathi[13] studied flow of fluid through a long uniform rectangular duct when impulsive pressure gradient acting at the central part of a section, Patel, Noorjahan and Kumar[7] studied MHD effect on free convective flow of a fluid through porous medium, Rapti[10] studied MHD flow in the presence of thermal radiation .

In the present paper our aim is to study the flow of a visco-elastic [Oldroyd (1958) type] liquid through porous medium and the annular space between two right circular cylinders when both cylinders oscillate harmonically with different amplitudes and frequencies. Some particular cases have also been discussed in detail.

*Basic Theory and Equations of Motion:*

For slow motion the rheological equations for Oldroyd (1958) visco-elastic liquid are :

$$\tau_{ij} = -p\delta_{ij} + \tau'_{ij}$$

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\tau'_{ij} = 2\mu\left(1+\mu_1 \frac{\partial}{\partial t}\right)e_{ij}$$

$$e_{ij} = \frac{1}{2}\left(\upsilon_{i,J} + \upsilon_{j,i}\right)$$

Where $\tau_{ij}$ = The stress tensor

$\tau'_{ij}$ = The deviatoric stress tensor

$e_{ij}$ = The rate of strain tensor

$p$ = The pressure

$\lambda_1$ = The stress relaxation time parameter

$\mu_1$ = The strain rate retardation time parameter

$\delta_{ij}$ = The metric tensor

$\mu$ = The coefficient of viscosity

$v_i$ = The velocity components

*Formulation of the Problem:*

Let (r, $\theta$, z) be the cylindrical polar coordinates and if $v_r, v_\theta, v_z$ are the components of velocity of liquid in the increasing directions of r, $\theta$ , and z – axis respectively. Consider the unsteady fully developed lamina flow of visco-elastic liquid through the porous medium and annular space between two right circular cylinders of radii a and b (a > b) with common axis as z – axis through porous medium. Assuming the pressure gradient to be zero.

For the present geometry the velocity components are $v_r = 0,\ v_\theta = 0,\ v_z = W(r,t)$ and appropriate equation of motion for Oldroyd (1958) visco-elastic liquid in presence of porous medium is given by:

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial W}{\partial t} = \upsilon\left(1+\mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial r^2}+\frac{1}{r}\frac{\partial W}{\partial r}\right)-\frac{1}{K}\left(1+\lambda_1 \frac{\partial}{\partial t}\right)W \tag{1}$$

Where r represents radius of cylinder, t the time, W the velocity of liquid along the axis of cylinder, $\upsilon = \frac{\mu}{\rho}$ = the kinetic viscosity, $\rho$ the density of liquid.

Since both the cylinders execute longitudinal harmonic oscillations along their common axis with different amplitudes and frequencies, therefore boundary conditions are :

$$\left.\begin{aligned} W &= V_1 e^{-i\omega_1 t} \qquad \text{when } r = a \\ W &= V_2 e^{-i\omega_2 t} \qquad \text{when } r = b \end{aligned}\right\} \tag{2}$$

where $V_1, \omega_1$ *and* $V_2, \omega_2$ are the respective amplitudes and frequencies of the outer and inner cylinders.

Introducing the following non-dimensional quantities:

$$r^* = \frac{r}{a}\ ,\quad t^* = \frac{\upsilon}{a^2}t\ ,\quad W^* = \frac{a}{\upsilon}W\ ,\quad \lambda_1^* = \frac{\upsilon}{a^2}\lambda_1\ ,\quad \mu_1^* = \frac{\upsilon}{a^2}\mu_1\ ,\quad V_1^* = \frac{a}{\upsilon}V_1\ ,$$

$$V_2^* = \frac{a}{\upsilon}V_2\ ,\ \omega_1^* = \frac{a^2}{\upsilon}\omega_1\ ,\ \omega_2^* = \frac{a^2}{\upsilon}\omega_2$$

in (1) and (2) and then dropping the stars, we get

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial W}{\partial t} = \left(1+\mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial r^2}+\frac{1}{r}\frac{\partial W}{\partial r}\right)-\frac{1}{K}\left(1+\lambda_1 \frac{\partial}{\partial t}\right)W \tag{3}$$

and boundary conditions

$$\left.\begin{aligned} W &= V_1 e^{-i\omega_1 t} \qquad \text{when } r = 1 \\ W &= V_2 e^{-i\omega_2 t} \qquad \text{when } r = b/a \end{aligned}\right\} \tag{4}$$

*Solution of the Problem :*

we look for a solution of equation (3) in the form

$$W = V_1 f(r)\, e^{-i\omega_1 t} + V_2\, g(r) e^{-i\omega_2 t} \tag{5}$$

which is evidently periodic in t.

Substituting (5) in (3), we get

$$\left[\left(1-i\,\omega_1\,\mu_1\right)\left\{\frac{d^2 f}{d r^2}+\frac{1}{r}\,\frac{d f}{d r}\right\}+\left\{i\,\omega_1\left(1-i\,\omega_1\,\lambda_1\right)-\frac{\left(1-i\,\omega_1\,\lambda_1\right)}{K}\right\}f\left(r\right)\right]V_1\,e^{-i\omega_1 t}$$

$$+\left[\left(1-i\,\omega_2\,\mu_1\right)\left\{\frac{d^2 g}{d r^2}+\frac{1}{r}\,\frac{d g}{d r}\right\}+\left\{i\,\omega_2\left(1-i\,\omega_2\,\lambda_1\right)-\frac{\left(1-i\,\omega_2\,\lambda_1\right)}{K}\right\}g\left(r\right)\right]V_2\,e^{-i\omega_2 t}=0 \qquad (6)$$

By assumption that $V_1$ and $V_2$ are not zero

We have $$\frac{d^2 f}{d r^2}+\frac{1}{r}\,\frac{d f}{d r}+\left(m_1^2-\frac{n_1^2}{r^2}\right)f=0 \qquad (7)$$

and $$\frac{d^2 g}{d r^2}+\frac{1}{r}\,\frac{d g}{d r}+\left(m_2^2-\frac{n_2^2}{r^2}\right)g=0 \qquad (8)$$

where $$m_1^2=\frac{\left(\lambda_1\,\omega_1^2+i\,\omega_1\right)}{\left(1-i\,\omega_1\,\mu_1\right)}$$

$$n_1^2=\frac{\left(1-i\,\omega_1\,\lambda_1\right)}{\left(1-i\,\omega_1\,\mu_1\right)K}$$

$$m_2^2=\frac{\left(\lambda_1\,\omega_2^2+i\,\omega_2\right)}{\left(1-i\,\omega_2\,\mu_1\right)}$$

$$n_2^2=\frac{\left(1-i\,\omega_2\,\lambda_1\right)}{\left(1-i\,\omega_2\,\mu_1\right)K}$$

Now boundary conditions (4) become

$$\left.\begin{array}{ll} f(r)=1,\quad g(r)=0 & when\ r=1 \\ f(r)=0,\quad g(r)=1 & when\ r={}^{b}\!/\!_{a} \end{array}\right\} \qquad (9)$$

Assuming that $m_1$ and $m_2$ are neither integers nor zero, the solutions of the equations (7) and (8) subject to boundary conditions (9) are:

$$f\left(r\right)=\frac{J_{-n_1}\left(m_1\frac{b}{a}\right)J_{n_1}\left(m_1 r\right)-J_{n_1}\left(m_1\frac{b}{a}\right)J_{-n_1}\left(m_1 r\right)}{J_{n_1}\left(m_1\right)J_{-n_1}\left(m_1\frac{b}{a}\right)-J_{n_1}\left(m_1\frac{b}{a}\right)J_{-n_1}\left(m_1\right)} \qquad (10)$$

$$g(r) = \frac{J_{-n_2}\left(m_2 \frac{b}{a}\right) J_{n_2}(m_2 r) - J_{n_2}\left(m_2 \frac{b}{a}\right) J_{-n_2}(m_2 r)}{J_{n_2}(m_2) J_{-n_2}\left(m_2 \frac{b}{a}\right) - J_{n_2}\left(m_2 \frac{b}{a}\right) J_{-n_2}(m_2)} \tag{11}$$

Putting the values of $f(r)$ and $g(r)$ in (5) we get the velocity of visco-elastic [Oldroyd (1958) type] liquid between two oscillating co-axial right circular cylinders through porous medium

$$W = V_1 \left\{ \frac{J_{-n_1}\left(m_1 \frac{b}{a}\right) J_{n_1}(m_1 r) - J_{n_1}\left(m_1 \frac{b}{a}\right) J_{-n_1}(m_1 r)}{J_{n_1}(m_1) J_{-n_1}\left(m_1 \frac{b}{a}\right) - J_{n_1}\left(m_1 \frac{b}{a}\right) J_{-n_1}(m_1)} \right\} e^{-i\omega_1 t} +$$

$$V_2 \left\{ \frac{J_{-n_2}\left(m_2 \frac{b}{a}\right) J_{n_2}(m_2 r) - J_{n_2}\left(m_2 \frac{b}{a}\right) J_{-n_2}(m_2 r)}{J_{n_2}(m_2) J_{-n_2}\left(m_2 \frac{b}{a}\right) - J_{n_2}\left(m_2 \frac{b}{a}\right) J_{-n_2}(m_2)} \right\} e^{-i\omega_2 t} \tag{12}$$

*Particular Cases :*
*Case – I*

If both cylinders of annular tube oscillate with same amplitudes but different frequencies, then $V_1 = V_2 = V$ (say) and from (12), we get

$$W = V \left[ \left\{ \frac{J_{-n_1}\left(m_1 \frac{b}{a}\right) J_{n_1}(m_1 r) - J_{n_1}\left(m_1 \frac{b}{a}\right) J_{-n_1}(m_1 r)}{J_{n_1}(m_1) J_{-n_1}\left(m_1 \frac{b}{a}\right) - J_{n_1}\left(m_1 \frac{b}{a}\right) J_{-n_1}(m_1)} \right\} e^{-i\omega_1 t} + \right.$$

$$\left. \left\{ \frac{J_{-n_2}\left(m_2 \frac{b}{a}\right) J_{n_2}(m_2 r) - J_{n_2}\left(m_2 \frac{b}{a}\right) J_{-n_2}(m_2 r)}{J_{n_2}(m_2) J_{-n_2}\left(m_2 \frac{b}{a}\right) - J_{n_2}\left(m_2 \frac{b}{a}\right) J_{-n_2}(m_2)} \right\} e^{-i\omega_2 t} \right] \tag{13}$$

*Case – II :*

If both cylinders of annular tube oscillate with same frequencies but different amplitudes, then $\omega_1 = \omega_2 = \omega$ (say) and from (12), we get

$$W = \left\{ \frac{J_{-n}\left(m\frac{b}{a}\right)J_n(mr) - J_n\left(m\frac{b}{a}\right)J_{-n}(mr)}{J_n(m)J_{-n}\left(m\frac{b}{a}\right) - J_n\left(m\frac{b}{a}\right)J_{-n}(m)} \right\}\left(V_1 + V_2\right)e^{-i\omega t} \tag{14}$$

*as* $m_1^2 = m_2^2 = m^2 \ (say)$ *and* $n_1^2 = n_2^2 = n^2 \ (say)$

*Case – III :*

If both cylinders of annular tube oscillate with same amplitudes and same frequencies, then $V_1 = V_2 = V$ (say) and $\omega_1 = \omega_2 = \omega$ (say) and from (12), we get

$$W = \left\{ \frac{J_{-n}\left(m\frac{b}{a}\right)J_n(mr) - J_n\left(m\frac{b}{a}\right)J_{-n}(mr)}{J_n(m)J_{-n}\left(m\frac{b}{a}\right) - J_n\left(m\frac{b}{a}\right)J_{-n}(m)} \right\} 2V\, e^{-i\omega t} \tag{15}$$

*Case – IV :*

If Porous medium is withdrawn i.e. K→∞, then $n_1^2 = n_2^2 = 0$ and from (7) and (8), we get

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} + m_1^2 f = 0 \tag{16}$$

and $$\frac{d^2 g}{dr^2} + \frac{1}{r}\frac{dg}{dr} + m_2^2 g = 0 \tag{17}$$

The solutions of (16) and (17) subject to the boundary conditions (9) are

$$f(r) = \frac{J_0(m_1 r)Y_0\left(m_1\frac{b}{a}\right) - Y_0(m_1 r)J_0\left(m_1\frac{b}{a}\right)}{J_0(m_1)Y_0\left(m_1\frac{b}{a}\right) - J_0\left(m_1\frac{b}{a}\right)Y_0(m_1)} \tag{18}$$

and $$g(r) = \frac{Y_0(m_2 r)J_0(m_2) - Y_0(m_2)J_0(m_2 r)}{J_0(m_2)Y_0\left(m_2\frac{b}{a}\right) - Y_0(m_2)J_0\left(m_2\frac{b}{a}\right)} \tag{19}$$

from (5), (18) and (19) we get velocity of visco-elastic liquid between two oscillating co-axial right circular cylinders

$$W = V_1\left\{ \frac{J_0(m_1 r)Y_0\left(m_1\frac{b}{a}\right) - Y_0(m_1 r)J_0\left(m_1\frac{b}{a}\right)}{J_0(m_1)Y_0\left(m_1\frac{b}{a}\right) - Y_0(m_1)J_0\left(m_1\frac{b}{a}\right)} \right\} e^{-i\omega_1 t} +$$

$$V_2 \left\{ \frac{J_0(m_2)Y_0(m_2 r) - Y_0(m_2\ )J_0(m_2 r)}{J_0(m_2)Y_0\left(m_2 \frac{b}{a}\right) - Y_0(m_2)J_0\left(m_2 \frac{b}{a}\right)} \right\} e^{-i\omega_2 t} \quad (20)$$

*Case – V :*

If we take $\lambda_2 = 0$ in above results, we get all results for velocity of Maxwell liquid.

*Case – VI :*

If we take $\lambda_1 = 0,\ \lambda_2 = 0$ in above results, we obtain all results for velocity of purely viscous liquid.

**Conclusion**

The presence of porous medium in the annular space between right circular cylinders affects the velocity of visco-elastic liquid.

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**References**

1. Arya, M. K.; Kumar, R. and Singh, K. K., International Journal of Mathematical Archiv, Vol. 4, No. 1, p-261 (2013).
2. Chakraborty, G. and Sengupta, P.R., Uzechoslovak Jour. Of physics Vol. 42, No. 5, p.-525 (1992).
3. Choubey, K.R., Ind. Jour. Pure Appl. Math., Vol.19, No. 8, p.-931 (1985).
4. Krishna, D.V. and Rao, P.N., Acta Ciencia Indica, Vol. XXXIM, No. 3, p.-917 (2005).
5. Krishna, D.V., Rao, P.N. and Sulochana, P., Bull. Pure. Appl. Sci., Vol. 23, No. 2, p.-291(2004).
6. Oldroyd, J.G., Proc. Roy. Soc. (London) A, 245, p.-278 (1958).
7. Patel, M., Noorjahan and Kumar, M. V., International journal of mathematical Archive, Vol.11 No. 7, p-31 (2020).
8. Radhakrishnamacharya, G. and Rao, S., Proc. Nat. Acad. Sci. India, Vol. 76 (A), No. III, p.-241(2007).
9. Rahman, M.M. and Alam Sarkar, M.S., Bull. Cal. Math. Soc., Vol. 96, No. 6, p.-463 (2004).
10. Rapti, E., International journal of mathematical Archive, Vol.11 No.11 p-16 (2020).
11. Rivlin, R.S. and Ericksen, J.L., Jour. Rational Mech. Anal, Vol. 4, p.-323 (1955).
12. Teipel, I., Acta Mech. Vol. 39, p.-277 (1981).
13. Tripathi, A., International journal of mathematical Archive, Vol. 7 No. 11, p-1 (2016).
14. Venkateswariu, B. and Naryana, P.V.S.,Walailak, J. Sci. & Tech., Vol. 12, No. 9, p-775 (2015).
15. Yadav, B.S. and Singh, K.K., Proc. Nat. Acad. Sci. India., Vol. 60 (A), No.1, p.-103 (1990).