ISSN 2231-346X (Print)
JUSPS-A Vol. 32(5), 29-36 (2020). Periodicity-Monthly
Section A
(Online) ISSN 2319-8044

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Mathematics
website:- www.ultrascientist.org
Estd. 1989

Zagreb-K-Banhatti index of a graph

V. R. KULLI[1] and B. CHALUVARAJU[2]

[1]Department of Mathematics, Gulbarga University, Gulbarga - 585 106 (India)
E-mail: vrkulli@gmail.com
[2]Department of Mathematics, Bangalore University, Jnana Bharathi Campus,
Bangalore -560 056 (India)
Corresponding Author E-mail: bchaluvaraju@gmail.com
http://dx.doi.org/10.22147/jusps-A/320501

Acceptance Date 06th August, 2020, Online Publication Date 17th August, 2020

Abstract

The Zagreb indices were introduced by Gutman and Trinajstic in 1972. The K-Banhatti indices were introduced by Kulli in 2016. These two types of indices are closely related. In this study, we define the Zagreb-K-Banhatti index of a graph. We establish some relations between Zagreb, K-Banhatti and Zagreb-K-Banhatti indices. We also obtain lower and upper bounds for the Zagreb -K-Banhatti index of a graph in terms of Zagreb and K- Banhatti indices.

Keywords: Zagreb index, K- Banhatti indices, Zagreb-K-Banhatti index.
2010 (AMS) Mathematics Subject Classification: 05C05, 05C07, 05C35.

1. Introduction

We consider only finite, simple graphs. Let $G = (V, E)$ be a connected graph with vertex set $V(G)$ and edge set $E(G)$. A graph G has n vertices and m edges. The degree $d_G(u)$ of a vertex is the number of vertices adjacent to u. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denote the degree of an edge $e = uv$ in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. The vertices and edges of a graph G are called the elements of G. For undefined terms and notations, we refer the book[1].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of Mathematical chemistry, which has an important effect on the development of the Chemical Sciences. In Chemical Science, physico-chemical properties of chemical compounds often modelled by means of molecular graph based structure descriptors, which are also referred to as topological indices or graph indices. For graph indices[2]. A single number that can be used to characterize some property of the underlying molecule is said to be a topological indices. Numerous such topological indices have been considered as Theoretical Chemistry, and have found some applications, especially in QSPR/QSAR/QSTR study[3-5].

The first and second Zagreb indices were introduced by Gutman *et al*[6]. They are defined as $M_1(G) = \sum_{u\in V(G)} d_G(u)^2$ and $M_2(G) = \sum_{uv\in E(G)}[d_G(u).d_G(v)]$. An alternative expression for the first Zagreb index[7] is $M_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]$.

Shirdel[8] *et al.*, introduced the first hyper-Zagreb index of a graph G and defined it as $HM_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]^2$.

Milicevic[9] *et al.*, introduced the first and second reformulated Zagreb indices of a graph G in terms of edge degrees instead of vertex degrees and defined as $EM_1(G) = \sum_{e\in E(G)} d_G(e)^2$ and $EM_2(G) = \sum_{e\sim f}[d_G(e).d_G(f)]$, where $e\sim f$ means that the edges e and f are adjacent. The third reformulated Zagreb index of a graph G is defined by $EM_3(G) = \sum_{e\sim f}[d_G(e) + d_G(f)]$.

Furtula[10] *et al.*, introduced the forgotten topological index of a graph G, defined as $F(G) = \sum_{uv\in E(G)}(d_G(u))^2 + (d(_G(v))^2 = \sum_{v\in V(G)} d_G(v)^3$.

Kulli[11] introduced the first and second K-Banhatti indices, intending to take into account the contributions of pairs of incident elements. The first and second K-Banhatti indices of a graph G are defined as $B_1(G) = \sum_{ue} d_G(u) + d_G(e)$ and $B_2(G) = \sum_{ue} d_G(u).d_G(e)$, respectively, where ue means that the vertex u and edge e are incident in G.

The first and second K hyper Banhatti indices were introduced by Kulli[12] and they are defined as $HB_1(G) = \sum_{ue}[d_G(u) + d_G(e)]^2$ and $HB_2(G) = \sum_{ue}[d_G(u).d_G(e)]^2$.

The Zagreb and K-Banhatti indices have been studied extensively. Their various modifications have been introduced. These two types of indices are closely related. For their history, applications, and mathematical properties[13-22] and the references cited therein.

Motivated by the work on Zagreb and K- Banhatti indices, we introduce the Zagreb-K-Banhatti index of a graph G and defined it as

$$MB(G) = \sum_{\substack{a \text{ is either adjacent} \\ \text{or} \\ \text{incident to } b}} [d_G(a) + d_G(b)],$$

where a and b are elements of G.

In this paper, we obtain some relations between the Zagreb-K-Banhatti index, Zagreb, K-Banhatti indices. Also, we provide lower and upper bounds for $MB(G)$ of a graph in terms of Zagreb –type indices.

2. Comparison of Zagreb-K-Banhatti, Zagreb, Banhatti-type indices :

We use the following result to prove Theorem 2.

Theorem 1[21]. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$B_1(G) = 3M_1(G) - 4m.$$

Theorem 2. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$MB(G) = 4M_1(G) + EM_3(G) - 4m.$$

Proof. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$MB(G) = \sum_{\substack{a \text{ is either adjacent} \\ \text{or} \\ \text{incident to } b}} [d_G(a) + d_G(b)]$$

$$= \sum_{ab \in E(G)} [d_G(a) + d_G(b)]$$

$$+ \sum_{e,f \in E(G),\ e \sim f} [d_G(a) + d_G(b)]$$

$$+ \sum_{a(ab)} [d_G(a) + d_G(b)]$$

$$= M_1(G) + EM_3(G) + B_1(G).$$

Using Theorem 1, we have $MB(G) = 4M_1(G) + EM_3(G) - 4m$.

We use the following theorem to prove our next result.

Theorem 3[21]. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$B_1(G) = HM_1(G) - EM_1(G) - M_1(G).$$

Theorem 4. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$MB(G) = HM_1(G) + EM_3(G) - EM_1(G).$$

From Theorem 2, we have

$$MB(G) = M_1(G) + EM_3(G) + B_1(G)$$

$$= HM_1(G) + EM_3(G) - EM_1(G).$$ Using Theorem 3.

To prove the next result, we use the earlier established;

Theorem 5[21]. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$B_1(G) = HM_1(G) + M_1(G) - B_2(G) - 4m.$$

Theorem 6. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$MB(G) = 2M_1(G) + HM_1(G) + EM_3(G) - B_2(G) - 4m.$$

Proof. From Theorem 2, we have

$$MB(G) = M_1(G) + EM_3(G) + B_1(G)$$
$$= 2M_1(G) + HM_1(G) + EM_3(G) - B_2(G) - 4m, \text{ by Theorem 5.}$$

We use the following result to prove Theorem 8.

Theorem 7 [21]. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$B_1(G) = \frac{1}{2} HB_1(G) - EM_1(G) + M_1(G) - 12m.$$

Theorem 8. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$MB(G) = 2M_1(G) + \frac{1}{2} HB_1(G) + EM_3(G) - EM_1(G) - 12m.$$

Proof . From Theorem 2, we have

$$B(G) = M_1(G) + EM_3(G) + B_1(G)$$
$$= 2M_1(G) + \frac{1}{2} HB_1(G) + EM_3(G) - EM_1(G) - 12m, \text{ by Theorem 7.}$$

We use the following result to establish our next result.

Theorem 9 [21]. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$B_1(G) = F(G) + 2M_2(G) - 4M_1(G) + 4m.$$

Theorem 10. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$MB(G) = F(G) + 2M_2(G) - 3M_1(G) + EM_3(G) + 4m.$$

Proof. From Theorem 2, we have

$$MB(G) = M_1(G) + EM_3(G) + B_1(G)$$
$$= F(G) + 2M_2(G) - 3M_1(G) + EM_3(G) + 4m, \text{ by Theorem 9.}$$

3. Bounds on Zagreb-K-Banhatti, Zagreb, K-Banhatti-type indices :

We use the following result to prove Theorem 12.

Theorem 11 [21]. For any connected graph G,

$$M_1(G) \leq B_1(G).$$

Equality is attained if and only if G is regular.

Theorem 12. Let G be a connected graph with $n \geq 3$ vertices and m edges. Then

$$2M_1(G) + EM_3(G) \leq MB(G).$$

Equality is attained if and only if G is regular.

Proof. From Theorem 2, we have $MB(G) = M_1(G) + EM_3(G) + B_1(G)$.

Using Theorem 11, we obtain $2M_1(G) + EM_3(G) \leq MB(G)$.

Second part is obvious.

We use the following results to prove our next result.

Theorem 13 [22]. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$\frac{4m^2}{n} \leq M_1(G).$$

Theorem 14 [21]. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$\frac{4m(3m-n)}{n} \leq B_1(G) \leq 3m^2 - m.$$

Theorem 15. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$\frac{4m(4m-n)}{n} + EM_3(G) \leq MB(G) \leq EM_3(G) + 4m^2.$$

Proof. From Theorem 2, we have $MB(G) = M_1(G) + EM_3(G) + B_1(G)$.
From Theorems 13 and 14, bearing in mind that $M_1(G) \leq m(m+1)$, we have

$$\frac{4m^2}{n} + EM_3(G) + \frac{4m(3m-n)}{n} \leq MB(G) \leq m(m+1) + EM_3(G) + 3m^2 - m.$$

Thus, $\frac{4m(4m-n)}{n} + EM_3(G) \leq MB(G) \leq EM_3(G) + 4m^2$.

We use the following result to prove our next result.

Theorem 16 [21]. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$2m[3\delta(G) - 2] \leq B_1(G) \leq 2m[3\Delta(G) - 2].$$

Further, equality in both lower and upper bounds is attained if and only if G is regular.

Theorem 17. Let G be a simple graph with $n \geq 3$ vertices and m edges. Then

$$4m[2\delta(G) - 1] + EM_3(G) \leq MB(G) \leq EM_3(G) + 4m[2\Delta(G) - 1].$$

Further, equality in both lower and upper bounds is attained if and only if G is regular.

Proof. We have $2\delta(G) \leq d_G(u) + d_G(v) \leq 2\Delta(G)$.

Thus $2m\,\delta(G) \leq M_1(G) \leq 2m\,\Delta(G)$. (1)

From Theorem 2, we have $MB(G) = M_1(G) + EM_3(G) + B_1(G)$.
From Theorem 16 and inequality (1), we have

$$4m[2\delta(G) - 1] + EM_3(G) \leq MB(G) \leq EM_3(G) + 4m[2\Delta(G) - 1].$$

Second part is obvious.

We use the following results to prove our next result.

Theorem 18 [15]. For any tree T with $n \geq 3$ vertices and m edges,

$$4n - 6 \leq M_1(T) \leq n(n-1).$$

Theorem 19 [21]. For any tree T with $n \geq 3$ vertices and m edges,

$$8n - 14 \leq B_1(T) \leq (n-1)(3n-4).$$

Theorem 20. For any tree T with $n \geq 3$ vertices and m edges,

$$4(3n-5) + EM_3(T) \leq MB(T) \leq EM_3(T) + 4(n-1)^2.$$

Proof. From Theorem 2, we have $MB(T) = M_1(T) + EM_3(T) + B_1(T)$.

From Theorems 18 and 19, we have

$(4n-6) + EM_3(T) + (8n-14) \leq MB(T) \leq n(n-1) + EM_3(T) + (n-1)(3n-4)$.
Therefore, $4(3n-5) + EM_3(T) \leq MB(T) \leq EM_3(T) + 4(n-1)^2$.

We use the following result to establish our next result in terms of minimum and maximum degree of a graph.

Theorem 21 [21]. For any connected graph G with $n \geq 3$ vertices and m edges,

$$B_1(G) \leq \frac{3n}{4}[\Delta(G) - \delta(G)]^2 + \frac{4m}{n}(3m - n).$$

Theorem 22. For any connected graph G with $n \geq 3$ vertices and m edges,

$$MB(G) \leq EM_3(G) + 2m\Delta(G) + \frac{3n}{4}[\Delta(G) - \delta(G)]^2 + \frac{4m}{n}(3m - n).$$

Proof. From Theorem 2, we have $MB(G) = M_1(G) + EM_3(G) + B_1(G)$.
From Theorem 21 and inequality (1), we have

$$MB(G) \leq 2m\Delta(G) + EM_3(G) + \frac{3n}{4}[\Delta(G) - \delta(G)]^2 + \frac{4m}{n}(3m - n).$$

We use the following result to establish our next result.

Theorem 23 [21]. For any connected graph G with $n \geq 3$ vertices and m edges,

$$B_1(G) \geq \frac{24\, m\, \delta(G)\, \Delta(G)}{[\Delta(G) + \delta(G)]^2} F(G) - 4m.$$

Theorem 24. For any connected graph G with $n \geq 3$ vertices and m edges,

$$MB(G) \geq 2\, m\, (\delta(G) - 2) + \frac{24\, m\, \delta(G)\, \Delta(G)}{[\Delta(G) + \delta(G)]^2} F(G) + EM_3(G).$$

Proof. From Theorem 2, we have $MB(G) = M_1(G) + EM_3(G) + B_1(G)$.
From Theorem 23 and inequality (1), we have

$$MB(G) \geq 2\, m\delta(G) + \frac{24\, m\, \delta(G)\, \Delta(G)}{[\Delta(G) - \delta(G)]^2} F(G) + EM_3(G) - 4m.$$

Therefore, $MB(G) \geq 2\, m\, (\delta(G) - 2) + \frac{24\, m\, \delta(G)\, \Delta(G)}{[\Delta(G) - \delta(G)]^2} F(G) + EM_3(G)$.

The following result is used to prove our next result.

Theorem 25 [21]. For any connected graph G with $n \geq 3$ vertices and m edges,

$HM_1(G) - M_1(G)(2\Delta(G)-1) + 4m(\Delta(G)-1) \leq B_1(G) \leq HM_1(G) - M_1(G) - 4(\delta(G)-1)^2$.

Theorem 26. For any connected graph G with $n \geq 3$ vertices and m edges,

$HM_1(G) - 2M_1(G)(\Delta(G)-1) + 4m(\Delta(G)-1) + EM_3(G) \leq MB(G) \leq HM_1(G) - 4(\delta(G)-1)^2 + EM_3(G)$.

Proof. From Theorem 2, we have $MB(G) = M_1(G) + EM_3(G) + B_1(G)$.

From Theorem 25, we have

$M_1(G) + HM_1(G) - M_1(G)(2\Delta(G)-1) + 4m(\Delta(G)-1) + EM_3(G) \leq MB(G) \leq M_1(G) + HM_1(G) - M_1(G) - 4(\delta(G)-1)^2$
$+ EM_3(G)$.

Therefore, $HM_1(G) - 2M_1(G)\,(\Delta(G)-1) + 4m\,(\Delta(G)-1) + EM_3(G) \leq MB(G) \leq HM_1(G) - 4\,(\delta(G)-1)^2 + EM_3(G)$.

We use the following result to prove our next result.

Theorem 27 [21]. For any (n, m)-graph G with p-pendant vertices and minimal non-pendant vertex degree $\delta_1(G)$,

$$6\,\delta_1(G)\,(m-p) + 3p(1+\delta_1(G)) - 4m \leq B_1(G) \leq 6\,(\Delta(G)\,(m-p) + 3p\,(1+(\Delta(G)) - 4m.$$

Theorem 28. For any (n, m)-graph G with p-pendant vertices and minimal non-pendant vertex degree $\delta_1(G)$,

$$2m\,\delta(G) + 6\,\delta_1(G)\,(m-p) + 3p\,(1+\delta_1(G)) - 4m + EM_3(G) \leq MB(G) \leq 2m\Delta(G) + 6\,(\Delta(G)\,(m-p) + 3p\,(1+(\Delta(G)) - 4m + EM_3(G).$$

Proof. From Theorem 2, we have $MB(G) = M_1(G) + EM_3(G) + B_1(G)$.

From Theorem 27 and inequality (1), we have

$$2m\delta(G) + 6\,\delta_1(G)\,(m-p) + 3p\,(1+\delta_1(G)) - 4m + EM_3(G) \leq MB(G) \leq 2m\Delta(G) + 6\,(\Delta(G)(m-p) + 3p\,(1+(\Delta(G)) - 4m + EM_3(G).$$

Conclusion

In this study, we have introduced the Zagreb-K-Banhatti index of a graph. We have obtained some relations between this new topological index with Zagreb and K-Banhatti indices. Further, we have established lower and upper bounds for $MB(G)$ of a connected graph in terms of Zagreb-Type indices. For the comparative advantages, applications and mathematical point of view, many questions are suggested by this research, among them are the following:

1. Find the extremal values and extremal graphs of the Zagreb -K-Banhatti index.
2. Characterize the Zagreb-K-Banhatti index in terms of other degree based topological indices.
3. Explores some results towards QSPR/QSAR/QSTR Model by using the Zagreb-K-Banhatti index.

References

1. V. R. Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).
2. V. R. Kulli, Graph Indices, in Hand Book of Research on Advanced Applications of Graph Theory in Modern Society, M. Pal, S. Samanta and A. Pal (eds.) IGI Global, USA 66-91 (2020).
3. I. Gutman, O. E. Polansky, Mathematical Concepts in Organic Chemistry, Springer, Berlin (1986).
4. V. R. Kulli, Multiplicative connectivity Indices of nanostructures, LAP LEMBERT Academic Publishing (2018).
5. R. Todeschini and V. Consonni, Molecular Descriptors for Chemoinformatics, Wiley–VCH, Weinheim (2009).
6. I. Gutman and N. Trinajstic, Graph Theory and molecular orbitals. Total -electron energy of alternant hydrocarbons, Chem. Phys. Lett. *17,* 535–538 (1972).
7. S. Nikolic, G. Kovacevic, A. Milicevic and N. Trinajastic, The Zagreb indices 30 years after, Croat. Chem. Acta., *76,* 113-124 (2003).
8. G. H. Shirdel, H. Rezapour and A. M. Sayadi, The hyper-Zagreb index of graph operations, Iran J. Math. Chem. *4(2),* 213-220 (2013).
9. A. Milicevic, S. Nikolic and N. Trinajastic, On reformulated Zagreb indices, Mol. Divers, *8,* 393-399 (2004).
10. B. Furtula and I. Gutman, A forgotten topological index, J. Math. Chem. *53,* 1184-1190 (2015).
11. V. R. Kulli, On K Banhatti indices of graphs, Journal of Computer and Mathematical Sciences, *7,* 213-218 (2016).

12. V. R. Kulli, On K hyper Banhatti indices and coindices of graphs, International Research Journal of Pure Algebra, *6(5),* 300-304 (2016).
13. B. Borovicanin, K. C. Das, B. Furtula and I. Gutman, Zagreb indices: Bounds and extremal graphs, in I. Gutman, B. Furtula, K.C. Das, E. Milovanovic and I. Milovanovic (eds.) Bounds in Chemical Graph Theory-Basics (pp. 67-153), Univ. Kragujevac, Kragujevac (2017).
14. I. Gutman, Degree based topological indices, Croat. Chem. Acta., *86,* 351-361 (2013).
15. I. Gutman and K. C. Das, The first Zagreb indices 30 years after, MATCH Commun. Math. Comput. Chem. *50,* 83-92 (2004).
16. I. Gutman, B. Furtula, Z. Kovijanic Vukicevic and G. Popivoda, Zagreb indices and coindices, MATCH Commun. Math. Comput. Chem., *74,* 5-16 (2015).
17. V. R. Kulli, On K Banhatti indices and K hyper Banhatti indices of V-Phenylenic nanotubes and nanotorus, Journal of Computer and Mathematical Sciences, *7(6),* 302-307 (2016).
18. V. R. Kulli, Computing Banhatti indices of network, International Journal of Advances in Mathematics, *1,* 31-40 (2018).
19. V. R. Kulli, K Banhatti indices of chloroquine and hydroxychloroquine: Research Applied for the treatment and prevention of COVID-19, SSRG International Journal of Applied Chemistry, *7(1),* 63-68 (2020).
20. V. R. Kulli, B. Chaluvaraju and H. S. Boregowda, K Banhatti and K hyper Banhatti indices of windmill graphs, South East Asain J. of Math. And Math. Sci. *13(1),* 11-18 (2017).
21. I. Gutman, V. R. Kulli, B. Chaluvaraju and H. S. Boregowda, On Banhatti and Zagreb indices, Journal of the International Mathematical Virtual Institute, *7,* 53-67 (2017).
22. B. Zhou and N. Trinajstic, Some properties of the reformulated Zagreb indices, J. Math. Chem., *48,* 714-719 (2010).