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# Thermo-Fluid Mechanics of gas-Solid Particle Flows over horizontal Flat Plate

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**Abstract**

Mathematical model has been developed to protect fluid and solid particle homogeneous mixture velocity concentration and temperature for a heated horizontal flat plate. Conversation equation based on Eulerian scale are approximated for small relaxation times through stream function and similarity transformations. Parametric database generated through computer program for arbitrary constants on comparison with clear fluid reveals the particle concentration has pronounced effect on velocity and temperature profiles.

*Key words :* Fluid Mechanics, Boundary layer equations, ordinary differential equation, Integro-differential equation

AMS Subject Classification: 35Q35 37N10, 76A04, 78M15.

## Introduction

**V**ery few publications have been written[1,3,4,12] on the boundary layer analysis of two-phase flows to protect mechanics of particles effects on Eulerian scale through integral methods. Because of the complexity of two phase turbulent boundary layer few mathematical models[5,6] have been started under laminar flow conditions, whose solutions are based on parametric expansion, which are not only radius but also restricted to particular conditions and cannot be extended to 2D or 3D geometrical problems. However this difficulties can be minimized with the increasing speed of computing digital devices through advanced numerical techniques. To this effect, few attempted[7] have been made for diffrent floor condition to understand the phenomena qualitatively.

The initial velocity profiles[9,10] for 1 to *m*, magnesia particles flowing through gas around flat plate is studied graphically. It has been considered a thin boundary layer of gas-particles around the plate, having it's surface parallel to the x- axis and normal to y-axis. The plate is heated and aligned with a uniform flow of constant velocity. The heat generated within the boundary layer due to frictional forces will be diffused a way from fluid element by thermal conductivity and at the same time, is convicted downstream by fluid motion. Since the plate is originally at the same temperature as that of the fluid for upstream, it will heated it up and hence surface temperature finally approach to a constant value when the the steady state is reached. Because, in a boundary layer fluid medium of gas-particles decelerates through irreversible process, the temperature at wall where velocity gradient is zero, is not the stagnation temperature of the free stream.

2 *Boundary layerequations :*

For constant physical properties[2,8,11], equation of gas-solid particles in steady state for the range $x < u/\tau$ from the leading edge are

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-f\left[u'\frac{\partial u'}{\partial x}+v'\frac{\partial u'}{\partial y}\right] \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{Pc_p}\left[\frac{\partial u}{\partial y}\right]^2-q\left[u'\frac{\partial T'}{\partial x}+v'\frac{\partial T'}{\partial y}\right] \tag{2.3}$$

$$u'\frac{\partial f}{\partial x}+f\frac{\partial u'}{\partial y}+v'\frac{\partial f}{\partial x}+v\frac{\partial f}{\partial y}=0 \tag{2.4}$$

The equations 2.1 to 2.4 are subjected to

$$u = u_0 = v = v' = \frac{\partial T}{\partial x}=\frac{\partial T'}{\partial y}=0;\quad f=f_0 \ \text{at}\ y=0$$

$$u = u' = u_0;\ T = T' = T_0 \quad \text{at}\ \ y=\infty \tag{2.5}$$

The equation of *y* component of fluid momentum is dropped by boundary layer simplification. Since $v' = O[v]$, consistency requires that the equation of *y* component momentum of particle fluid interaction also dropped. Particle relaxation and thermal relaxation times are defined as

$$\tau=\frac{6\pi\mu a}{m}\ \ and\ \ \tau'=\frac{4\pi k a}{mc_3} \tag{2.6}$$

when

$$\frac{\tau}{\tau_0} < 1,\ u = u' \ \text{and}\ \ v = v' \tag{2.7}$$

$$\frac{\tau'}{\tau_0} < 1 \; T = T' \tag{2.8}$$

Assumptions

- It is observed that boundary layer thickness is small compared with the characteristic length of the body *l*.
- The dynamic and thermal boundary that exists at $Pr < 1$ for the fluid considered are in equalorder.
- Temperatures in system are within a range approximately $1000^oF$ that all radiation effects are neglected.
- The fluid particle medium that exist over the plate is considered laminar, stable and steady state.
- The size of particle is spherical of type fine in the range 1-15$\mu m$ suspended in fluid, whose effects over time are assumed negligible.

3 *Formulation :*

In view of above assumption and condition 2.7, equations 2.2 and 2.3 gives rise to

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{u}}{\partial y} = 0 \tag{3.1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = \frac{v}{1+f}\frac{\partial^2 \bar{u}}{\partial y^2} \tag{3.2}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \frac{1}{Pr[1+q]}\frac{\partial^2 \bar{T}}{\partial y^2} + \frac{v}{k[1+q]\Delta T}\left[\frac{\partial \bar{u}}{\partial y}\right]^2 \tag{3.3}$$

$$\bar{u}\frac{\partial f}{\partial x} + f\frac{\partial u}{\partial y} + \bar{v}\frac{\partial f}{\partial x} + f\frac{\partial f}{\partial y} = 0 \tag{3.4}$$

are subject to

$$\bar{u} = \bar{v} = \frac{\partial \bar{T}}{\partial y} = 0; \;\; f = f_0 \text{ at } y = 0$$

$$\bar{u} = \bar{u}_0; \;\; \bar{T} = \bar{T}_0 \qquad \text{at } y = \infty \tag{3.5}$$

*4 Similarity transformations :*

In order to convert the equations 3.1 and 3.2 in to ordinary differential equation, define the stream function $\psi$ such that

$$u = \frac{\partial \psi}{\partial y}; \; v = \frac{-\partial \psi}{\partial x} \tag{4.1}$$

on substitution, equation 3.2 will be rearranged as

$$\frac{\partial^2 \psi}{\partial y^2}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \frac{v}{1+f}\frac{\partial^3 \psi}{\partial y^3} \tag{4.2}$$

Using following transformation and non dimensional quantities

$$\psi = [vxu_0]^{1/2} c(\eta);\; Pr = \frac{\mu cp}{k};\; Br = \frac{\mu}{k\Delta T};\; f = \frac{mN}{\rho}$$

$$\eta = y\left[\frac{u_0}{xv}\right]^{1/2};\; \theta = \frac{2[T - T_1]c_p}{u_0^2};\; q = f - \frac{c_s}{c_p} \tag{4.3}$$

The governing equation to describ fleuid particle velocity temperature and particle concentration are

$$\frac{d^3c}{d\eta^3} + \frac{1+f}{2}\frac{d^3c}{d\eta^3} = 0 \tag{4.4}$$

$$\frac{d^2\theta}{d\eta^2} + \frac{cPr(1+q)}{2}\frac{d\theta}{d\eta} + 2Pr\,Br\left[\frac{dc'}{d\eta}\right]^2 = 0 \tag{4.5}$$

$$\frac{df'}{d\eta} = \frac{2c'}{\eta c' - c} \tag{4.6}$$

$$\eta = 0;\; c = \frac{dc}{d\eta} = 0;\; \theta = 0;\; f' = f_0$$

$$\eta = 0;\; c = 0;\; \theta = \theta_0 \tag{4.7}$$

*5 Analysis :*

In the equation 4.4, if $c, c'$ and $c''$ are known at non dimensional height $\eta_i$, the fourth order Runge-kutta method has been employed to find the solution at $\eta_{i+1} = L+h$ and stations there after step by step. In order to use this method equation 4.4 are written into three first order equations as

$$\frac{dc}{d\eta} = (1+f)p;\; \frac{dp}{d\eta} = q;\; \frac{dq}{d\eta} = cq/2 \tag{5.1}$$

However, the numerical integration of equation 5.1 cannot be started at $\eta = 0$ because $q$ is not know there. Boundary conditions 4.7 provide only two or three values of $c$ and $c'$ that are required at $\eta = 0$ but provide another value of $p$ it infinity. Therefore, in order to solve this problem by Ranga-kutta method, it has to be combined with half interval search method.

The boundary conditions 4.7 at infinity is not convenient for computations, since every value specied in the problem must be finite. Instead of extending to infinity, it was limited range of numerical integration upto a reasonable large heigh $\eta_{max}$ and approximated by

$$1 - p < \in \quad \text{at } \eta = \eta_{max} \tag{5.2}$$

Where $\in$ represents positive value much less than unity, whose magnitude controls the accuracy of solution. With this scheme on computations of $c$, $c'$ and $c''$ the parallel and normal velocity components are related as

$$\frac{\bar{u}}{u_0} = c' \tag{5.3}$$

$$v = \left[\frac{v u_0}{x}\right]^{1/2} \frac{\eta c' - c}{2} \tag{5.4}$$

Using the values of $c$ and $c'$ particle concentration over each location of horizontal plate is determined Runge-Kutta Method through the algorithm defined as

$$f_{i+1} = f_1 + \frac{1}{2}\,[\Delta_1 f_1 + 2\,\Delta_2 f_2 + 2\,\Delta_3 f_3 + \Delta_4 f_4] \tag{5.5}$$

The temperature quantity involved in the equation 4.5 is second order linear differential equation, whose coefficients contain tabulated functions of $c$ and numerically $c'$. This equation has solved through finite difference wherein the axis of finite independent has been divided into small intervals of variable width.

Let $\theta$ be such neigh boring with $j = i + m$ where $m$ is positive or negative integer. For small value of $h$, the function evaluated at $\theta$ through Taylor series about $\theta$ give rise to

$$c_{i_1}\,\theta_{i-1} + c_{i_2}\theta_i + c_{i_3}\,\theta_{i+1} = c + i_4$$

where

$$c_{i_1} = 1 - \frac{h}{4} Pr(1+q), \quad c_{i_3} = 1 + \frac{h}{4} Pr(1+q)$$

$$c_{i_2} = -2h^2 \qquad c_{i_4} = 2Br\,Prh(c'')^2$$

In which, third term is known from the boundary condition at $\eta_{max}$. This boundary condition is incorporated in the numerical scheme by

$$c_{n_4} < -(c_{n_4} - c_{n_3}\,\theta_{n+1}), \qquad c_{n_3} < 0$$

Having modified four coefficients, $c_{i_j}$, the solution of $\theta$ is computed by solving $n$ simultaneous equations arranged in the triangulation matrix.

## 6 **Discussion of Results**

Figure 6 shows the distribution pattern for air and air-particle mixture for different concentration.

the component $u$ for clean air approaches free stream velocity ($u_0$) following smooth curve in 31 iterations. If the boundary layer thickness defined as height where $u = 0.994u_0$ which layer is parabola. when the through air, it's solution convergence in 36 iterations, whose plot indicates considerable changes in magnitudes of air-particle mixture velocity.

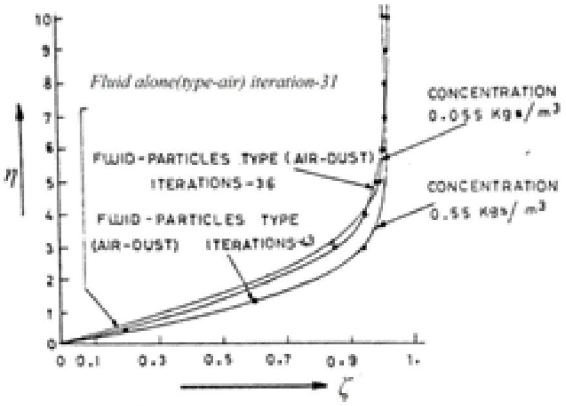


Figure 6.1: Velocity profiles of fluid and two-phase particles over horizontal plate.

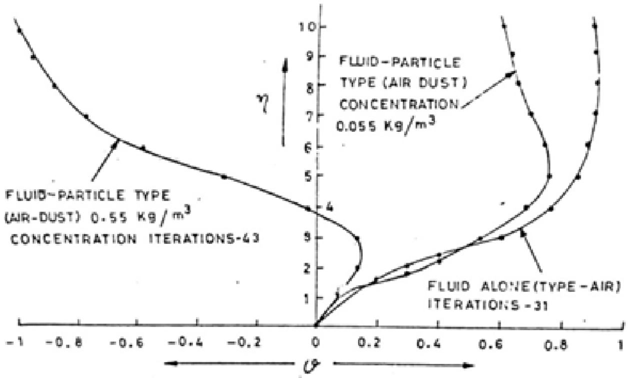


Figure 6.2: Normal velocity pro les of uid and two-phase uid -particles over horizontal plate.

The solution of $v$ for clean air which converges the corresponding iterations approaches constant value when going away from plate to free stream velocity.

The phenomena for dense particles in air explains the same pattern in the plate. But for more dense particles, the component $v$ goes negative values after attaining positive velocity at $\eta = 3.8$.

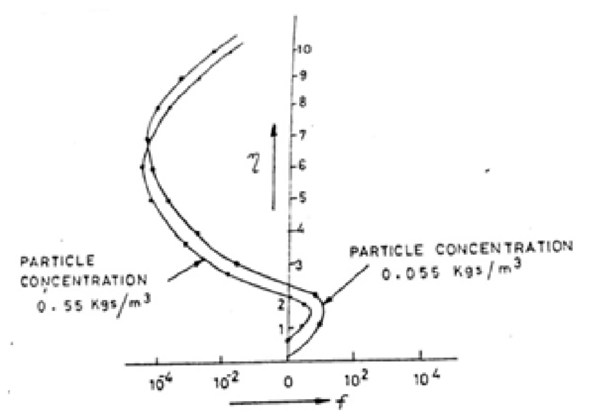


Figure 6.3: Particle Concentration distribution

Particle density distribution as a function of *u* and *v* are computed and it's magnitudes over are plotted for different initial concentration in 6.3. Due to presence of velocity components, concentration distribution for little dense particles shows considerable effect up to certain distance of plate and afterwards becomes small. The temperature variations of air and air-particles in 6 reveals that the magnitudes for dense and more dense particles are smaller than clean fluid and hence reaches constant values very fast.

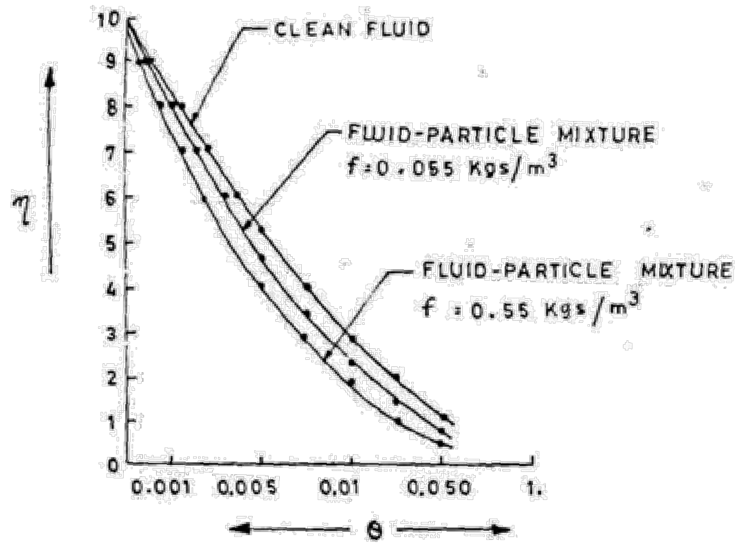


Figure 6.4: Temperature Pro les

## 7 **Conclusion**

The presence of dense particles, which creates deviation in profile paths have the effect on boundary layer growth. However the effect is more significant when particle concentration in flowing air is of order 0.55 $kg/m^3$. Under adiabatic conditions temperature variations of air particle mixture reveals loss of energy due to changes in thermal properties.

## 8 *Scope ofresearch :*

Energy transfer between parallel plates with buoyancy forces are of fundamental importance to heat ex-changers whose applications are pivotal in process and power generation equipment. Problems of this nature associated with gas-solid flows provide insight to understand FAILURES of systems due to surface EROSION. The techniques for evaluating free convection effects caused by arranging vertical plates in rows and columns are also important to electronics industry.

For example, in many cases, micro-electronic components are mounted in substrates (plates) that dissipate energy, they generate by free convection, electrical and space requirements reveals that these substrates be stacked closed to gather leading to thermal interactions between them. When the heated plates are stacked in columns, the plate valves interacts with boundary layers. Such interactions could produce results signicantly, when the fluid medium is subjected to two phases. Before going to general analysis for columns and rows of substrates, a sound theoretical knowledge of two phase, flow between heated iso-thermal plates are highly desirable.

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