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Preserving Neutrosophic Feebly Closed SetsK. BAGEERATHI¹ and P. JEYA PUVANESWARI²¹Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur - 628216 (India)²Department of Mathematics, Vivekananda College, Agasteeswaram – 629701 (India)Corresponding Author Email : mrcsm5678@rediffmail.com , jeypuvaneswari@gmail.com<http://dx.doi.org/10.22147/jusps-A/310501>

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Abstract

In this article, the concept of neutrosophic feebly homeomorphism in neutrosophic topological spaces is introduced. Further, the work is extended as almost neutrosophic feebly totally open mappings, almost neutrosophic feebly totally continuous functions, super neutrosophic feebly clopen continuous functions in neutrosophic topological spaces and establishes some of their related attributes.

Keywords and Phrases: neutrosophic feebly homeomorphism, neutrosophic topological space, almost neutrosophic feebly totally open mappings, almost neutrosophic feebly totally continuous functions and super neutrosophic feebly clopen continuous functions.

AMS Subject Classification: 03E72, 06D72**Introduction**

The concept of fuzzy set was introduced by Zadeh¹⁸ in 1965. Since then fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy set uses one real value $\mu_A(x) \in [0, 1]$ to represent the grade of membership of fuzzy set A defined on universe X . In some applications such as expert system, belief system and information fusion, we should consider not only the truth membership supported by the evident but also the falsity membership against by the evident. In 1986, Atanassov² introduced the intuitionistic fuzzy set which is a generalization of fuzzy sets. The intuitionistic fuzzy set consider both truth membership and falsity membership $\sigma_A(x)$, with $\mu_A(x), \sigma_A(x) \in [0, 1]$ and $0 \leq \mu_A(x) + \sigma_A(x) \leq 1$.

Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and

inconsistent information which exists commonly in belief system. In Intuitionistic fuzzy sets, $1 - \mu_A(x) - \sigma_A(x)$ by default.

For example, when we ask the opinion of an expert about certain statement, he or she says that the possibility that the statement is true is 0.8 and the statement is false is 0.9 and the degree that he or she is not sure is 0.4. In neutrosophic set, indeterminacy is quantified explicitly and truth membership, in determinacy membership and falsity membership are independent. This assumption is very important role in a lot of situations such as information fusion when we try to combine the data for different sensors.

Neutrosophy was introduced by Smarandache⁵⁻⁸ in 1995. "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra". Neutrosophic set is a power general formal frame work which generalizes the concept of the classic set, fuzzy set, intuitionistic fuzzy set etc. In 2014, Salama, Smarandache and Valeri¹⁴ were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions. The neutrosophic homeomorphism was introduced by Parimala *et al.*¹³.

Iswarya *et al.*⁹ defined the concept of neutrosophic semi open sets in neutrosophic topological spaces. Jeya puvaneswari *et al.*¹⁰⁻¹² defined neutrosophic feebly open sets, neutrosophic feebly closed sets and neutrosophic feebly continuous functions in neutrosophic topological spaces.

In this paper, the concept of neutrosophic feebly homeomorphism in neutrosophic topological spaces is introduced. Further, the work is extended as almost neutrosophic feebly totally open mappings, almost neutrosophic feebly totally continuous functions, super neutrosophic feebly clopen continuous functions in neutrosophic topological spaces and establishes some of their related attributes.

This paper is organized as follows. Section II gives the basic definitions of neutrosophic feebly open sets & closed sets, neutrosophic feebly continuous functions in neutrosophic topological spaces and their properties which are used in the later sections. The Section III deals with the concept of neutrosophic feebly homeomorphism. Section IV explains almost neutrosophic feebly totally open mappings in neutrosophic topological spaces and their properties. Section V shows that the concept of almost neutrosophic feebly totally continuous functions. In section VI, super neutrosophic feebly clopen continuous functions in neutrosophic topological spaces and some of their related properties are studied.

II. Preliminaries :

First we shall present the fundamental definitions. The following one is obviously inspired by Smarandache⁵⁻⁸ and Salama¹⁴⁻¹⁶.

*Definition 2.1.*¹⁶ Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A .

Neutrosophic sets 0_N and 1_N in X as follows:

0_N may be defined as:

$$(0_1) \ 0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$$

$$(0_2) \ 0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$$

$$(0_3) \ 0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$$

$$(0_4) \ 0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$$

1_N may be defined as:

$$(1_1) 1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$$

$$(1_2) 1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$$

$$(1_3) 1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$$

$$(1_4) 1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$$

*Definition 2.2.*¹⁶ Let $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$ be a neutrosophic set in X . Then the complement of the set A [A^C for short] may be defined as three kinds of complements:

$$(C_1) A^C = \{ \langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \gamma_A(x) \rangle : x \in X \}$$

$$(C_2) A^C = \{ \langle x, \gamma_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$$

$$(C_3) A^C = \{ \langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$$

One can define several relations and operations between neutrosophic sets follows:

*Definition 2.3.*¹⁶ Let be a non-empty set, and neutrosophic sets A and B in the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle : x \in X \}$. Then we may consider two possible definitions for subsets ($A \leq B$).

$A \leq B$ may be defined as:

$$(1) A \leq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x), \forall x \in X$$

$$(2) A \leq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x), \forall x \in X$$

*Definition 2.4.*¹⁶ Let be a non-empty set, and $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle, B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ are neutrosophic subsets. Then

(1) $A \wedge B$ may be defined as:

$$(I_1) A \wedge B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x) \text{ and } \gamma_A(x) \vee \gamma_B(x) \rangle$$

$$(I_2) A \wedge B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x) \text{ and } \gamma_A(x) \vee \gamma_B(x) \rangle$$

(2) $A \vee B$ may be defined as:

$$(U_1) A \vee B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x) \text{ and } \gamma_A(x) \wedge \gamma_B(x) \rangle$$

$$(U_2) A \vee B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x) \text{ and } \gamma_A(x) \wedge \gamma_B(x) \rangle$$

*Definition 2.5.*¹⁶ A neutrosophic topology [NT for short] is a non-empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms:

$$(NT_1) 0_N, 1_N \in \tau,$$

$$(NT_2) G_1 \wedge G_2 \in \tau \text{ for any } G_1, G_2 \in \tau,$$

$$(NT_3) \bigvee G_i \in \tau \text{ for every } \{G_i : i \in J\} \subseteq \tau$$

In this case the pair (X, τ) is called a neutrosophic topological space. The elements of τ are called neutrosophic open sets.

*Definition 2.6.*¹⁶ The complement of A [A^C for short] of neutrosophic open sets is called a neutrosophic closed set in X .

*Definition 2.7*¹⁴ Let X and Y be two nonempty neutrosophic sets and $f: X \rightarrow Y$ be a function.

(i) If $B = \{ (y, \mu_B(y), \sigma_B(y), \gamma_B(y)) : y \in Y \}$ is a Neutrosophic set in Y , then the pre image of B under f is denoted and defined by $f^{-1}(B) = \{ (x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)) : x \in X \}$.

(ii) If $A = \{ (x, \alpha_A(x), \delta_A(x), \lambda_A(x)) : x \in X \}$ is a NS in X , then the image of A under f is denoted and defined by $f(A) = \{ (y, f(\alpha_A)(y), f(\delta_A)(y), f(\lambda_A)(y)) : y \in Y \}$ where $f(\lambda_A) = C(f(C(A)))$.

In (i), (ii), since $\mu_B, \sigma_B, \gamma_B, \alpha_A, \delta_A, \lambda_A$ are neutrosophic sets, we explain that $f^{-1}(\mu_B)(x) = \mu_B(f(x))$,

$$\text{and } f(\alpha_A)(y) = \begin{cases} \sup \{ \alpha_A(x) : x \in f^{-1}(y) \}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

*Lemma 2.8*¹⁴ Let $f: X \rightarrow Y$ be a function. If A is a neutrosophic subset of X and μ is a neutrosophic subset of Y . Then

- (i) $f(f^{-1}(A)) \leq A$
- (ii) $f(f^{-1}(A)) = A \Leftrightarrow f$ is surjective.
- (iii) $f^{-1}(f(A)) \geq A$
- (iv) $f^{-1}(f(A)) = A$ whenever f is injective.

*Definition 2.9*¹⁰ A neutrosophic subset A of a neutrosophic topological space (X, τ) is neutrosophic feebly open if there is a neutrosophic open set U in X such that $U \leq A \leq NScl(U)$.

*Lemma 2.10*¹⁰ (i) Every neutrosophic open set is a neutrosophic feebly open set.

*Lemma 2.11*¹⁰ A neutrosophic subset A of a neutrosophic topological space (X, τ) is neutrosophic feebly closed if and only if $Ncl(Nint(Ncl(A))) \leq A$.

*Definition 2.11*¹¹ Let (X, τ) and (Y, σ) be neutrosophic topological spaces. Then a map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called neutrosophic continuous (in short N-continuous) function if the inverse image of every neutrosophic open set in (Y, σ) is neutrosophic open set in (X, τ) .

*Definition 2.12*¹² Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. A function $f: X \rightarrow Y$ is called neutrosophic feebly irresolute if the inverse image of every neutrosophic feebly open set in Y is neutrosophic feebly open in X .

*Lemma 2.13*¹² Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. A function $f: X \rightarrow Y$ is neutrosophic feebly irresolute if and only if the inverse image of every neutrosophic feebly closed set in Y is neutrosophic feebly closed in X .

*Lemma 2.14*¹² Every neutrosophic feebly irresolute function is neutrosophic feebly continuous.

*Definition 2.15*¹⁰ Let $\alpha, \beta, \gamma \in [0, 1]$ and $\alpha + \beta + \gamma \leq 1$. A Neutrosophic point with support $x_{(\alpha, \beta, \gamma)} \in X$ is a

neutrosophic set of X is defined by $x_{(\alpha, \beta, \gamma)} = \begin{cases} (\alpha, \beta, \gamma), & y = x \\ (0, 0, 1), & y \neq x \end{cases}$

In this case, x is called the support of $x_{(\alpha, \beta, \gamma)}$ and α, β and γ are called the value, intermediate value and the non – value of $x_{(\alpha, \beta, \gamma)}$ respectively. A Neutrosophic point $x_{(\alpha, \beta, \gamma)}$ is said to belong to a neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ is denoted by two way (i) $x_{(\alpha, \beta, \gamma)} \in A$ if $\alpha \leq \mu_A(x), \beta \leq \sigma_A(x)$ and $\gamma \geq \gamma_A(x)$. (ii) $x_{(\alpha, \beta, \gamma)} \in A$ if $\alpha \leq \mu_A(x), \beta \geq \sigma_A(x)$ and $\gamma \geq \gamma_A(x)$.

Clearly a neutrosophic point can be represented by an ordered triple of Neutrosophic set as follows : $x_{(\alpha, \beta, \gamma)} = (x_\alpha, x_\beta, C(x_{C(\gamma)}))$. A class of all neutrosophic points in X is denoted as $NP(X)$.

*Definition 2.16.*¹² Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. A function $f: X \rightarrow Y$ is said to be neutrosophic feebly closed if the image of each neutrosophic closed set in X is neutrosophic feebly closed in Y .

*Definition 2.17.*¹² Let (X, τ) and (Y, σ) be two neutrosophic topological spaces. A function $f: X \rightarrow Y$ is said to be neutrosophic feebly open if the image of each neutrosophic open set in X is neutrosophic feebly open in Y .

*Lemma 2.18.*¹² For any bijective map $f: X \rightarrow Y$ the following statements are equivalent:

- (i) $f^{-1}: Y \rightarrow X$ is neutrosophic feebly continuous.
- (ii) f is neutrosophic feebly open.
- (iii) f is neutrosophic feebly closed.

III. Neutrosophic Feebly Homeomorphism :

The purpose of this section is to introduces the idea of neutrosophic feebly homeomorphism in neutrosophic topological spaces and establish some of their attributes.

Definition 3.1. Let X and Y be neutrosophic topological spaces. A mapping $f: X \rightarrow Y$ is said to be a neutrosophic feebly homeomorphism if f is bijective, neutrosophic feebly continuous and neutrosophic feebly open.

Example 3.2. Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$ and A and B be neutrosophic subset such that

$$A = \langle x, (0.5, 0.4, 0.3), (0.7, 0.8, 0.2) \rangle,$$

$B = \langle y, (0.1, 0.7, 0.6), (0.8, 0.9, 0.5) \rangle$. Then $\tau = \{0_N, 1_N, A\}$ and $\{0_N, 1_N, B\}$, are two neutrosophic topology over X and Y , respectively. Moreover, let $f: X \rightarrow Y$ be a function such that $f(x_1) = y_1$ and $f(x_2) = y_2$. It can be seen that f is a neutrosophic homeomorphism.

Theorem 3.3. Let (X, τ) and (Y, σ) be two neutrosophic topological spaces and $f: X \rightarrow Y$ be a bijective function. Then f is a neutrosophic feebly homeomorphism if and only if f is a neutrosophic feebly continuous and neutrosophic feebly closed function.

Proof. Let f is a neutrosophic feebly homeomorphism. From Definition 3.1 f is a neutrosophic feebly continuous function. From Lemma 2.18, we have f^{-1} is a neutrosophic feebly closed function. So, $(f^{-1})^{-1} = f$ is a neutrosophic feebly closed function.

Theorem 3.4. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. If g is neutrosophic feebly continuous, then the following statements are equivalent:

- a) g is a neutrosophic feebly closed mapping.
- b) g is a neutrosophic feebly open mapping.

c) g^{-1} is a neutrosophic feebly homeomorphism.

Proof. (a) \Rightarrow (b) Let us assume that g is a bijective mapping and a neutrosophic feebly closed mapping. Hence, g^{-1} is a neutrosophic feebly continuous mapping. Since each neutrosophic open set is a neutrosophic feebly open set, g is a neutrosophic feebly open mapping.

(b) \Rightarrow (c) Let g be a bijective and neutrosophic open mapping. Furthermore, g^{-1} is a neutrosophic feebly continuous mapping. Hence, g and g^{-1} are neutrosophic feebly continuous. Therefore, g is a neutrosophic feebly homeomorphism.

(c) \Rightarrow (a) Let g be a neutrosophic feebly homeomorphism. Then g and g^{-1} are neutrosophic feebly continuous. Since each neutrosophic closed set in X is a neutrosophic feebly closed set in Y , hence g is a neutrosophic feebly closed mapping.

IV. Almost Neutrosophic Feebly Totally Open Mappings :

In this section, we introduce almost neutrosophic feebly totally mappings and we discuss some basic properties.

Definition 4.1. A function $f: X \rightarrow Y$ is said to be

(i) Almost neutrosophic feebly open if the image of each neutrosophic feebly regular open set in X is neutrosophic feebly open in Y .

(ii) Almost neutrosophic feebly closed (briefly almost feebly closed) if the image of each neutrosophic feebly regular closed set in X is neutrosophic feebly closed in Y .

(iii) Almost neutrosophic feebly clopen if the image of each neutrosophic feebly regular clopen set in X is neutrosophic feebly clopen in Y .

(iv) neutrosophic feebly totally open if the image of each neutrosophic feebly open set in X is neutrosophic feebly clopen in Y .

(v) neutrosophic feebly totally closed if the image of each neutrosophic feebly closed set in X is neutrosophic feebly clopen in Y .

(vi) Almost neutrosophic feebly totally open if the image of each neutrosophic feebly regular open set in X is neutrosophic feebly clopen in Y .

(vii) Almost neutrosophic feebly totally closed if the image of each neutrosophic feebly regular closed set in X is neutrosophic feebly clopen in Y .

(viii) Almost neutrosophic feebly totally clopen if the image of each neutrosophic feebly regular clopen set in X is neutrosophic feebly clopen in Y .

Theorem 4.2. Every almost neutrosophic feebly totally closed map is almost neutrosophic feebly closed.

Proof. Let X and Y be neutrosophic topological spaces. Let $f: X \rightarrow Y$ be an almost neutrosophic feebly totally closed mapping. To prove f is almost neutrosophic feebly closed, let H be any neutrosophic feebly regular closed subset of X . Since f is almost neutrosophic feebly totally closed mapping, $f(H)$ is neutrosophic feebly clopen in Y . This implies that $f(H)$ is neutrosophic feebly closed in Y . Therefore f is almost neutrosophic feebly closed.

Corollary 4.3. Every neutrosophic feebly totally open map is almost neutrosophic feebly open.

Theorem 4.4. If a bijective function $f: X \rightarrow Y$ is almost neutrosophic feebly totally open, then the image

of each neutrosophic feebly regular closed set in X is neutrosophic feebly clopen in Y .

Proof. Let F be a neutrosophic feebly regular closed set in X . Then F^C is neutrosophic feebly regular open in X . Since f is almost neutrosophic feebly totally open, $f(F^C) = [f(F)]^C$ is neutrosophic feebly clopen in Y . This implies that $f(F)$ is neutrosophic feebly clopen in Y .

Theorem 4.5. A surjective function $f: X \rightarrow Y$ is almost neutrosophic feebly totally open if and only if for each subset B of Y and for each neutrosophic feebly regular open set U containing $f^{-1}(B)$, there is a neutrosophic feebly clopen set V of Y such that $B \leq V$ and $f^{-1}(V) \leq U$.

Proof. Suppose $f: X \rightarrow Y$ is a surjective and almost neutrosophic feebly totally open function and $B \leq Y$. Let U be neutrosophic feebly regular open set of X such that $f^{-1}(B) \leq U$. Since f is almost neutrosophic feebly totally open function, $f(U) = [f(U^c)]^C$ is neutrosophic feebly clopen. Then $V = [f(U^c)]^C$ is neutrosophic feebly clopen subset of Y containing B such that $f^{-1}(V) \leq U$.

Theorem 4.6. A map $f: X \rightarrow Y$ is almost neutrosophic feebly totally open if and only if for each subset A of Y and each neutrosophic feebly regular closed set U containing $f^{-1}(A)$ there is a neutrosophic feebly clopen set V of Y such that $A \leq V$ and $f^{-1}(V) \leq U$.

Proof. Suppose f is almost neutrosophic feebly totally open. Let $A \leq Y$ and U be a neutrosophic feebly regular closed set of X such that $f^{-1}(A) \leq U$. Now U^c is neutrosophic feebly regular open and f is almost neutrosophic feebly totally open, $f(U^c)$ is neutrosophic feebly clopen set in Y . Then $V = (f(U^c))^C$ is a neutrosophic feebly clopen set in Y . Note that $f^{-1}(A) \leq U$ implies $A \leq Y$ and $f^{-1}(V) = (f^{-1}(f(U^c)))^C \leq (U^c)^C = U$. That is $f^{-1}(V) \leq U$.

Conversely, let F be a neutrosophic feebly regular open set of X . Then $f^{-1}(f(F)^C) \leq F^C$ and F^C is neutrosophic feebly regular closed set in X . By hypothesis, there exist a neutrosophic feebly clopen set V in Y such that $f(F^C) \leq V$ and $V^C \leq f(F)$ and so $F \leq (f^{-1}(V))^C$. Hence $f(F) \leq f((f^{-1}(V))^C)$ which implies $f(F) \leq V^C$. Since V^C is neutrosophic feebly clopen, $f(F)$ is neutrosophic feebly clopen. That is $f(F)$ is neutrosophic feebly clopen in Y . Therefore f is almost neutrosophic feebly totally open.

Corollary 4.7. A map $f: X \rightarrow Y$ is almost neutrosophic feebly totally closed if and only if for each subset A of Y and each neutrosophic feebly regular open set U containing $f^{-1}(A)$, there is a neutrosophic feebly clopen set V of Y such that $A \leq V$ and $f^{-1}(V) \leq U$.

Theorem 4.8. If $f: X \rightarrow Y$ is almost neutrosophic feebly totally closed and A is neutrosophic feebly regular closed subset of X then $f_A: A \rightarrow Y$ is almost neutrosophic feebly totally closed.

Proof. Consider the function $f_A: A \rightarrow Y$ and let V be any neutrosophic feebly clopen set in Y . Since f is almost neutrosophic feebly totally closed, $f^{-1}(V)$ is neutrosophic feebly regular closed subset of X . Since A is feebly regular closed subset of X and $f_A^{-1}(V) = A \wedge f^{-1}(V)$ is neutrosophic feebly regular closed in A , it follows $f_A^{-1}(V)$ is neutrosophic feebly regular closed in A . Hence f_A is almost neutrosophic feebly totally closed.

Remark 4.9. Almost neutrosophic feebly totally clopen mapping is almost neutrosophic feebly open and almost neutrosophic feebly totally closed map.

V. Almost Neutrosophic Feebly Totally Continuous Functions :

In this section, some new continuous functions are introduced and discussed their characterizations.

Definition 5.1. A map $f: X \rightarrow Y$ is said to be

(i) neutrosophic feebly totally continuous if $f^{-1}(V)$ is neutrosophic feebly clopen in X for each neutrosophic feebly open set V in Y .

(ii) Almost neutrosophic feebly totally continuous if $f^{-1}(V)$ is neutrosophic feebly clopen in X for each neutrosophic feebly regular open set V in Y .

(iii) Almost neutrosophic feebly totally clopen continuous if $f^{-1}(V)$ is neutrosophic feebly clopen in X for each neutrosophic feebly regular clopen set V in Y .

Theorem 5.2. A function $f: X \rightarrow Y$ is almost neutrosophic feebly totally continuous function if the inverse image of every neutrosophic feebly regular closed set of Y is neutrosophic feebly clopen in X .

Proof. Let $f: X \rightarrow Y$ be almost neutrosophic feebly totally continuous function and F be any neutrosophic feebly regular closed set in Y . Then F^c is neutrosophic feebly regular open set in Y . Since f is almost neutrosophic feebly totally continuous, $f^{-1}(F^c)$ is neutrosophic feebly clopen in X . That is $(f^{-1}(F))^c$ is neutrosophic feebly clopen in X . This implies that $f^{-1}(F)$ is neutrosophic feebly clopen in X .

Theorem 5.3. A function $f: X \rightarrow Y$ is almost neutrosophic feebly totally continuous function is an almost neutrosophic feebly continuous function.

Proof. Suppose $f: X \rightarrow Y$ is almost neutrosophic feebly totally continuous and U is any neutrosophic feebly regular open subset of Y . Since f is almost neutrosophic feebly totally continuous, $f^{-1}(U)$ is feebly clopen in X . This implies that $f^{-1}(U)$ is neutrosophic feebly open in X . Therefore the function f is almost neutrosophic feebly continuous.

Theorem 5.4. For any bijective map $f: X \rightarrow Y$ the following statements are equivalent:

- (i) $f^{-1}: Y \rightarrow X$ is almost neutrosophic feebly totally continuous.
- (ii) f is almost neutrosophic feebly totally open.
- (iii) f is almost neutrosophic feebly totally closed.

Proof.

(i)→(ii): Let U be a neutrosophic feebly regular open set of X . By assumption, $(f^{-1})^{-1}(U) = f(U)$ is neutrosophic feebly clopen in Y and so f is almost neutrosophic feebly totally open.

(ii)→(iii): Let F be a neutrosophic feebly regular closed set of X . Then F^c is neutrosophic feebly regular open set in X . By assumption $f(F^c)$ is neutrosophic feebly clopen in Y . Hence f is almost neutrosophic feebly totally closed.

(iii)→(i): Let F be a neutrosophic feebly regular closed set of X . By assumption, $f(F)$ is neutrosophic feebly clopen set in Y . But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is almost neutrosophic feebly totally continuous.

VI. Super Neutrosophic Feebly Clopen Continuous Functions :

In this section, we introduce the concept of super neutrosophic feebly clopen continuous in neutrosophic topological spaces.

Definition 6.1. A map $f: X \rightarrow Y$ is said to be super neutrosophic feebly clopen continuous if for each $x_{(\alpha,\beta,\gamma)} \in X$ and for each neutrosophic feebly clopen set V containing $f(x_{(\alpha,\beta,\gamma)})$ in Y , there exist a neutrosophic feebly regular open set U containing $x_{(\alpha,\beta,\gamma)}$ such that $f(U) \leq V$.

Theorem 6.2. Let $f: X \rightarrow Y$ be almost neutrosophic feebly totally open. Then f is super neutrosophic feebly clopen continuous if $f(x_{(\alpha,\beta,\gamma)})$ is neutrosophic feebly clopen in Y .

Proof. Let G be neutrosophic feebly clopen in Y . Now $f^{-1}(G)$ is feebly regular open in X . Since the intersection of feebly clopen set is feebly clopen in Y , $f(f^{-1}(G)) = G \wedge f(x_{(\alpha,\beta,\gamma)})$ is feebly clopen in Y . Therefore, $f^{-1}(G)$ is feebly regular open in X . Hence f is super feebly clopen continuous function.

Theorem 6.3. If $f: X \rightarrow Y$ is surjective and almost neutrosophic feebly totally open, then f is super neutrosophic feebly clopen continuous.

Proof. Let G be neutrosophic feebly clopen in Y . Take $A = f^{-1}(G)$. Since $f(A) = G$ is neutrosophic feebly clopen in Y , by the Theorem 6.2, A is neutrosophic feebly regular open in X . Therefore f is super neutrosophic feebly clopen continuous.

Theorem 6.4. Let X, Y and Z be neutrosophic topological spaces. Then the composition $g \circ f: X \rightarrow Z$ is super neutrosophic feebly clopen continuous function where $f: X \rightarrow Y$ is super neutrosophic feebly clopen continuous function and $g: Y \rightarrow Z$ is neutrosophic feebly clopen irresolute function.

Proof. Let A be a neutrosophic feebly regular closed set of X . Since f is super neutrosophic feebly clopen continuous, $f(A)$ is neutrosophic feebly clopen in Y . Then by hypothesis, $f(A)$ is feebly clopen set. Since g is neutrosophic feebly clopen irresolute, $g(f(A)) = (g \circ f)(A)$. Therefore $g \circ f$ is super feebly clopen continuous.

Theorem 6.5. If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are two mappings such that their composition $g \circ f: X \rightarrow Z$ is almost neutrosophic feebly totally closed mapping then the following statements are true.

(i) If f is super neutrosophic feebly clopen continuous and surjective, then g is a neutrosophic feebly clopen irresolute function.

(ii) If g is neutrosophic feebly clopen irresolute function and injective, then f is an almost neutrosophic feebly totally closed function.

Proof. (i) Let A be a neutrosophic feebly clopen set of Y . Since f is super neutrosophic feebly clopen continuous, $f^{-1}(A)$ is neutrosophic feebly regular closed in X . Since $(g \circ f)(f^{-1}(A))$ is neutrosophic feebly clopen in Z . Since f is surjective, $g(A)$ is neutrosophic feebly clopen in Z . Therefore g is neutrosophic feebly clopen irresolute function.

(ii) Let B be neutrosophic feebly regular closed set of X . Since $g \circ f$ is almost neutrosophic feebly totally closed, $(g \circ f)(B)$ is neutrosophic feebly clopen set in Z . Since g is a neutrosophic feebly clopen irresolute function, $g^{-1}((g \circ f)(B))$ is neutrosophic feebly clopen set in Y . That is $f(B)$ is neutrosophic

feebly clopen set in Y . Since f is injective, f is an almost neutrosophic feebly closed function.

Conclusion

In this chapter, we have continued to study the properties of neutrosophic feebly Homeomorphism, almost feebly totally open mappings, almost feebly totally continuous functions, super neutrosophic feebly clopen continuous function and established the relations between them we obtain some new characterizations of these mappings and investigate preservation properties we expect the results in this chapter will be basis for further applications why of mappings in neutrosophic sets.

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