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Pie Generalizations-Locally Closed Sets

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Abstract

The aim of this paper is to continue the study of generalizations of locally closed sets and investigate the classes of πgl-continuous functions in a topological space.

Key words : *πglc, πg-open set, θ-locally closed set, πglc** and *πglc***

AMS Subject Classification (2010): 54A05

Introduction

The initiation of the study of generalized closed sets was done by Levine[6] in 1970. The notion of *πg*-closed sets as a weak form of generalized closed sets was introduced by Dontchev and Noiri[4] in 2000. The notion of locally closed sets in a topological space was introduced by Bourbaki[3]. Ganster and Reilly[5] further studied the properties of locally closed sets and defined the LC-continuity and LC-irresoluteness. Balachandran *et al.*[2] introduced the concepts of generalized locally. closed sets and GLC-continuous functions and investigated some of their properties. In 1997, Arockiarani *et al.*[1] studied regular generalized locally closed sets and RGL-continuous functions in a topological space.

The aim of this chapter is to continue the study of generalizations of locally closed sets and investigate the classes of *π*gl-continuous functions and *π*gl-irresolute functions in a topological space.

A set $A \subseteq (X, \tau)$ is called θ-closed [65] if $A = cl_\theta(A)$, where $cl_\theta(A) = \{x \in X : cl(U) \cap A = \phi, U \in \tau$ and $x \in U\}$. The complement of a θ-open set is called θ-closed. Before entering into our work, we recall the following definitions which are prerequisite for this paper.

Main Results

Definition 2.1

A subset S of (X, τ) is said to be πg-locally closed (πglc) if $S = G \cap F$ where G is πg-open and F is πg-closed in (X, τ).

Definition 2.2

A subset S of (X, τ) is called πglc^* if there exists a πg-open set G and a closed set F of (X, τ) such that $S = G \cap F$.

Definition 2.3 :

A subset B of (X, τ) is called πglc^{**} if there exists an open set G and a πg-closed set F of (X, τ) such that $B = G \cap F$.

The collection of all πg-locally closed (resp. πglc^*, πglc^{**}) sets of a space (X, τ) will be denoted by $\pi GLC(X, \tau)$ (resp. $\pi GLC^*(X, \tau)$, $\pi GLC^{**}(X, \tau)$).

From the above definitions we have the following remark.

Remark 2.4:

1. *Every locally closed set is πglc.*
2. *Every θ-locally closed set is πglc.*
3. *Every θglc-set is πglc.*
4. *Every πglc^*-set or πglc^{**} is πglc.*
5. *Every glc-set is πglc.*
6. *Every θlc-set is πglc^* or πglc^{**}.*
7. *Every glc^*-set is πglc^*.*
8. *Every θlc^*-set is πglc^*.*
9. *Every θlc^{**}-set is πglc^{**}.*
10. *Every θglc^*-set is πglc^*.*
11. *Every locally closed set is πglc^* and πglc^{**}.*

However the converses of the above are not true may be seen by the following Examples.

Example 2.5

Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then locally closed sets are ϕ, X, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d} and πglc-sets are P(X). It is clear that {a, c} is πglc-set but it is not locally closed.

Example 2.6

In the above Example 2.5, θ-locally closed sets are ϕ, X and πglc-sets are P(X). It is clear that {a, b} is πglc-set but it is not θ-locally closed set.

Example 2.7

In Example 2.5., θglc-sets are ϕ, X, {a},{b}, {a, b}, {c, d}, {a, c, d}, {b, c, d} and πglc-sets are P(X). It is clear that {b, c} is πglc-set but it is not θglc-set.

Example 2.8

Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, X, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$. Then πglc^*-sets are ϕ, X, {a}, {b}, {c}, {d}, {e}, {a, b}, {a, c}, {a, d}, {a, e}, {b, e}, {c, d}, {c, e}, {d, e}, {a, b,e}, {a, c, d}, {a, c, e}, {a, d, e}, {b, c, d}, {c, d, e}, {a, b, c, d}, {a, c, d, e},{b, c, d, e} and πglc-sets are P(X). It is clear that {b, *c} is πglc-set but it is not πglc^*-set.*

Example 2.9

In Example 2.5, πglc-sets are P(X) and glc-sets are ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}. It is clear that {b, c} is πglc-set but it is not glc-set.

Example 2.10

In Example 2.5, θlc-sets are ϕ, X and πglc^* (or) πglc^{**}-sets are P(X). It is clear that {a, b} is πglc^* (or) πglc^{**}-set but it is not θlc-set.

Example 2.11

In Example 2.5, glc^*-sets are ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d} and πglc^*-sets are P(X). It is clear that {b, c} is πglc^*-set but it is not glc^*-set.

Example 2.12

In Example 2.5, θlc^*-sets are ϕ, X, {c}, {d}, {c, d} and πglc^* sets are P(X). It is clear that {a, d} is πglc^*-set but it is not θlc^*-set.

Example 2.13

In Example 2.5, θlc^{**}-sets are ϕ, X, {a, b}, {a, b, c}, {a, b, d} and πglc^{**}-sets are P(X). It is clear that {a} is πglc^{**}-set but it is not θlc^{**}-set.

Example 2.14

In Example 2.5, θglc^*-sets are ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {c, d} and πglc^*-sets are P(X). It is clear that {b, c} is πglc^*-set but it is not θglc^*-set.

Example 2.15

In Example 2.5, locally closed sets are ϕ, X, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d} and πglc^* and πglc^{**}-sets are P(X). It is clear that {a, c} is both πglc^* and πglc^{**}-set but it is not locally closed set.

Theorem 2.16

For a subset S of (X, τ) the following are equivalent:

1. *$S \in \pi GLC^*(X, \tau)$.*
2. *$S = P \cap cl(S)$ for some πg-open set P.*
3. *$cl(S) - S$ is πg-closed.*
4. *$S - (X - cl(S))$ is πg-open.*

Proof.

(1) $\Rightarrow$ (2):

Let $S \in \pi GLC^*(X, \tau)$. Then there exists a πg-open set P and a closed set F such that $S = P \cap F$. Since $S \subset P$ and $S \subseteq cl(S)$ we have $S \subseteq P \cap cl(S)$.

Conversely, since $cl(S) \subseteq F$, $P \cap cl(S) \subseteq P \cap F = S$ which implies that $S = P \cap cl(S)$.

(2) $\Rightarrow$ (1):

Since P is πg-open and cl(S) is closed

$P \cap cl(S) \in \pi GLC^*(X, \tau)$.

$(3) \Rightarrow (4)$:

Let $F = cl(S)–S$. Then F is πg-closed by the assumption and $X–F = X \cap (cl(S)–S)^c = S \cup (X–cl(S))$. But X–F is πg-open. This shows that $S \cup (X–cl(S))$is πg-open.

$(4) \Rightarrow (3)$:

Let $U = S\cup (X–cl(S))$. Then U is πg-open. This implies that X–U is πg-closed and $X–U = X–(S\cup (X–cl(S))) = cl(S)\cap(X–S) = cl(S)–S$. Thus cl(S)–S is πg-closed.

$(4)\Rightarrow(2)$:

Let $U = S\cup (X–cl(S))$. Then U is πg-open. Hence we prove that $S = U \cap cl(S)$ for some πg-open set U. $U\cap cl(S) = (S \cup (X–cl(S))) \cap cl(S) = (cl(S) \cap S)\cup (cl(S)\cap X–cl(S)) = S\cup\phi = S$. Therefore $S = U\cap cl(S)$.

$(2) \Rightarrow (4)$:

Let $S = P\cap cl(S)$ for some πg-open set P. Then we prove that $S\cup(X–cl(S))$ is πg-open.
$S\cup (X–cl(S)) = (P\cap cl(S))\cup (X–cl(S)) = P\cap$
$(cl(S)\cup X–cl(S)) = P\cap X = P$ which is πg-open. Thus $S\cup (X–cl(S))$ is πg-open.

Definition 2.17

A topological space (X, τ) is called πg-submaximal (resp.g-submaximal[11]) if every dense subset is πg-open (resp. g-open).

Theorem 2.18

A topological space (X, τ) is πg-submaximal if and only if $P(X)= \pi GLC^*(X, \tau)$.

Proof.
Necessity:

Let $S\in P(X)$ and let $V = S\cup (X–cl(S))$.

Then V is πg-open and $cl(V) = cl(S)\cup (X–cl(S)) = X$. This implies that V is a dense subset of X. By the above Theorem $S\in\pi GLC^*(X,\tau)$. Therefore, $P(X) = \pi GLC^*(X,\tau)$.

Sufficiency:

Let S be a dense subset of (X,τ).

Then $S\cup (X–cl(S)) = S \Rightarrow S\in\pi GLC^*(X,\tau)$ and S is πg-open. This proves that X is πg-submaximal.

Remark 2.19.

It follows from definitions that if (X, τ) is g-submaximal, then it is πg-submaximal. But the converse is not true as seen by the following Example.

Example 2.20.

In Example 3., dense sets are X, {a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, g-open sets are ϕ, X, {a}, {b}, {a, b}, {a, b, c}, {a, b, d} and πg-open sets are P(X). Then it is πg-submaximal but not g-submaximal.

Theorem 2.21

For a subset S of (X, τ) if $S\in\pi GLC^{**}(X, \tau)$ then there exists an open set P such that $S = P\cap\pi g\text{-}cl(S)$ where $\pi g\text{-}cl(S)$ is the πg-closure of S.

Proof.

Let $S \in \pi GLC^{**}(X,\tau)$. Then there exists an open set P and a πg-closed set F such that $S = P \cap F$. Since $S \subseteq$ P and

$S \subseteq \pi g\text{-cl}(S)$, we have $S \subseteq P \cap \pi g\text{-cl}(S)$.

Conversely since $\pi g\text{-cl}(S) \subseteq F$, we have $P \cap \pi g\text{-cl}(S) \subseteq P \cap F = S$. Thus $S = P \cap \pi g\text{-cl}(S)$.

Theorem 2.22.

Let A and B be subsets of (X, τ). If $A \in \pi GLC^{*}(X,\tau)$ and $B \in \pi GLC^{*}(X, \tau)$ then $A \cap B \in \pi GLC^{*}(X, \tau)$.

Proof.

Let A and $B \in \pi GLC^{*}(X,\tau)$. Then there exist πg-open sets P and Q such that $A = P \cap cl(A)$ and $B = Q \cap cl(B)$. Therefore $A \cap B = P \cap cl(A) \cap Q \cap cl(B) = P \cap Q \cap cl(A) \cap cl(B)$ where $P \cap Q$ is πg-open and cl(A) and cl(B) is closed. This shows that $A \cap B \in \pi GLC^{*}(X,\tau)$.

Theorem 2.23.

If $A \in \pi GLC^{**}(X, \tau)$ and B is open, then $A \cap B \in \pi GLC^{**}(X,\tau)$.

Proof.

Let $A \in \pi GLC^{**}(X,\tau)$. Then there exists an open set G and a πg-closed set F such that $A = G \cap F$.

So $A \cap B = G \cap F \cap B = G \cap B \cap F$. This proves that $A \cap B \in \pi GLC^{**}(X, \tau)$.

Theorem 2.24

If $A \in \pi GLC(X, \tau)$ and B is πg-open, then A)" B" πGLC(X,Ä).

Proof.

Let $A \in \pi GLC(X,\tau)$. Then $A = G \cap F$ where G is πg-open and F is πg-closed. So $A \cap B = G \cap F \cap B = G \cap B \cap F$. This implies that $A \cap B \in \pi GLC(X,\tau)$.

Theorem 2.25

If $A \in \pi GLC^{*}(X, \tau)$ and B is πg-closed π-open subset of X, then $A \cap B \in \pi GLC^{*}(X, \tau)$.

Proof.

Let $A \in \pi GLC^{*}(X,\tau)$. Then $A = G \cap F$ where G is πg-open and F is closed. $A \cap B = G \cap (F \cap B)$ where G is πg-open and $F \cap B$ is closed. Hence $A \cap B \in \pi GLC^{*}(X, \tau)$.

Theorem 2.26

Let A and Z be subsets of (X, τ) and let $A \subseteq Z$. If Z is πg-open in (X, τ) and $A \in \pi GLC^{*}(Z, \tau/Z)$, then $A \in \pi GLC^{*}(X, \tau)$.

Proof.

Suppose A is πglc^{*}-set, then there exists a πg-open set G of $(Z,\tau/Z)$ such that $A = G \cap cl_Z(A)$. But $cl_Z(A) = Z \cap cl(A)$. Therefore, $A = G \cap Z \cap cl(A)$ where $G \cap Z$ is πg-open. Thus $A \in \pi GLC^{*}(X,\tau)$.

Example 2.27

Let $X = \{a, b, c, d, e\}$, $\tau = \{\phi, X, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$. Let V

be the collection of all π*g-open sets of* (X, τ). *Then* $V = \{\phi, X, \{a\}, \{c\}, \{d\}, \{e\}, \{a, c\}, \{a, d\}, \{a, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$. *Put* $Z = A = \{a, b, c\}$. *Then Z is not* π*g-open and* $A \in \pi glc*(Z, \tau /Z)$.
However $A \in \pi glc^*(X, \tau)$.

Theorem 2.28.

If Z is π*g-closed,* π*-open set in* (X, τ) *and* $A \in \pi GLC^*(Z, \tau/Z)$ *then* $A \cap \pi GLC*(X, \tau)$.

Proof.

Let $A \in \pi GLC^*(Z,\tau /Z)$. Then $A = G \cap F$ where G is πg-open and F is closed in $(Z,\tau/Z)$. Since F is closed in $(Z,\tau /Z)$, $F = B \cap Z$ for some closed set B of (X,τ). Therefore $A = G \cap B \cap Z$. Then $B \cap Z$ is closed. Hence $A \in \pi GLC*(X,\tau)$.

Theorem 2.29

If Z is closed and open in (X, τ) *and* $A \in \pi GLC(Z, \tau /Z)$, *then* $A \in \pi GLC(X, \tau)$.

Proof.

Let $A \in \pi GLC(Z,\tau /Z)$. Then there exists a πg-open set G and a πg-closed set F of $(Z,\tau /Z)$ such that $A = G \cap F$. Then by the above Theorem $A \in \pi GLC(X,\tau)$.

Theorem 2.30

If Z is π*g-closed,* π*-open subset of X and* $A \in \pi GLC^{**}(Z, \tau /Z)$, *then* $A \in \pi GLC^{**}(X, \tau)$.

Proof.

Let $A \in \pi GLC^{**}(Z,\tau /Z)$. Then $A = G \cap F$ where G is open and F is πg-closed in $(Z,\tau /Z)$. Since Z is πg-closed π-open subset of (X,τ), then F is πg-closed in (X,τ). Therefore $A \in \pi GLC^{**}(X,\tau)$.

Theorem 2.31

If A is π*g-open and B is open, then* $A \cap B$ *is* π*g-open.*

Proof.

Let A be πg-open. Then $int(A) \supseteq F$ whenever $A \supseteq F$ and F is π-closed set. Suppose $A \cap B \supseteq F$, then we prove that $int(A \cap B) \supseteq F$. Since B is open, $int(B) = B \supseteq F$. Therefore by assumptions $int(A \cap B) = int(A) \cap int(B) \supseteq F$. This proves that $A \cap B$ is πg-open.

Theorem 2.32

Suppose that the collection of all π*g-open sets of* (X, τ) *is closed under finite unions. Let* $A \in \pi GLC^*(X, \tau)$ *and* $B \in \pi GLC^*(X,\tau)$. *If A and B are separated, then* $A \cup B \in \pi GLC^*(X, \tau)$.

Proof.

Let $A, B \in \pi GLC^*(X,\tau)$. Then there exist πg-open sets G and S of (X,τ) such that $A = G \cap cl(A)$ and $B = S \cap cl(B)$. Put $V = G \cap (X - cl(B))$ and
$W = S \cap (X - cl(A))$. Then V and W are πg-open sets and $A = V \cap cl(A)$ and $B = W \cap cl(B)$. Also $V \cap cl(B) = \phi$ and $W \cap cl(A) = \phi$. Hence it follows that V and W are πg-open sets of (X,τ).
Therefore $A \cup B = (V \cap cl(A)) \cup (W \cap cl(B)) = V \cup W \cap cl(A) \cup cl(B)$.

Here $V \cup W$ is πg-open by assumption. Thus $A \cup B \in \pi GLC^{*}(X,\tau)$.

Example 2.33

Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\{a, b\}$ and $\{a, d\} \in \pi glc^{*}(X,\tau)$ but $\{a, b, d\} \in \pi glc^{*}(X, \tau)$, since they are not separated. For we have $\{a, b\} \cap cl(\{a, d\}) = \{a\} = \phi$ and $\{a, d\} \cap cl(\{a,b\}) = \{a, d\} = \phi$.

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