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Optimal Recruitment Policy for Manpower Planning Using Equilibrium Distribution

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Abstract

In manpower planning, one of the most important variables is duration until a specified event occurs. This is frequently the completed Length of service until leaving a job, which enables us to predict staff turnover. In manpower planning it is commonly the case that employees withdraw from active service for a period of time before returning to take up post at a later date. Such periods of absence are frequently of major concern to employers who are anxious that employees return as soon as possible. A model is derived in which the demand for manpower is assumed to be a random variable. Since the demand for manpower is not deterministic, it undergoes fluctuations. It may be noted that the demand distribution satisfies the so called Lack of Memory Property (LMP). Recently, there has been much attention given to higher order equilibrium distributions associated with a given distribution function. In this paper, a optimal recruitment policy has been discussed using Setting Clock Back to Zero(SCBZ) property using Equilibrium distribution. Numerical examples are also highlighted.

Key words : Manpower Planning, Reliability, Survival Analysis, Inventory.
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1.0. Introduction

Recently, there has been much attention given to higher order equilibrium distributions associated with a given distribution function, refer to Fagiuoli(1993,1994), Nanda *et.al.*(1996) and Li (2011). The equilibrium distributions has interest of researchers from various fields ever since it was introduced by Cox (1962), which sparked applications to numerous areas such as characterization of distributions, criteria for ageing, formulation

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of maintenance policies, income analysis, insurance etc.. In manpower planning, one of the most important variables is duration until a specified event occurs. This is frequently the completed Length of service until leaving a job, which enables us to predict staff turnover. In manpower planning it is commonly the case that employees withdraw from active service for a period of time before returning to take up post at a later date. Such periods of absence are frequently of major concern to employers who are anxious that employees return as soon as possible. A model is derived in which the demand for manpower is assumed to be a random variable. Since the demand for manpower is not deterministic, it undergoes fluctuations. It may be noted that the demand distribution satisfies the so called Lack of Memory Property (LMP). In this paper, a optimal recruitment policy has been discussed using Setting Clock Back to Zero(SCBZ) property using Equilibrium distribution. Numerical examples are also highlighted.

Several applications of equilibrium distributions include the areas of characterization of distributions by Gupta (1979), Hitha and Nair (1989), Nair and Hitha (1989), Gupta and Kirmani (1990), Sen Khattree (1996), etc. From perusal of the literature, it appears equilibrium distribution and its properties are most studied in the context of reliability modeling and analysis. Various aspects investigated in this respect can be summarized as follows. The relationships between various concepts in reliability for the equilibrium distribution and the baseline distribution are most important among them. These in turn provide the basis of many characterizations of lifetime models. Secondly most of the ageing concepts can be either interpreted or characterized by appropriate properties of the equilibrium distribution. Further, many new ageing concepts are evolved by comparing the ageing patterns of the baseline distribution and the corresponding equilibrium counterpart. Equilibrium distributions of higher orders have been proposed by a process of iteration that brings in new models whose characteristics can be expressed in terms of the original model. Many such relationships provide new methodology for establishing simple proofs in several cases and also enable statistical inference and analysis. The role of equilibrium distributions is fundamental in deriving proofs of properties of stochastic orders connecting reliability functions.

2.0. Equilibrium Distribution :

Some classifications of reliability distributions are based on properties of higher order equilibrium distributions. Although much attention has been paid to the equilibrium distributions associated with a given d.f., most results are for continuous random variables.

Let X be a random variable admitting an absolutely continuous distribution function F(x) with respect to the Lebesgue measure in the support of the set of non-negative real and having a finite mean μ. Associated with X, a new random variable Y is defined, whose p.d.f. is

$$f(x) = \frac{S(x)}{\mu}, \quad x>0 \tag{1}$$

where $S(x) = P(X \geq x)$ is the survival function of X.

The probabilistic comparison of Y with parent population of X is utilized to explain the phenomenon of ageing. Gupta (1984) obtained the equilibrium distribution as a weighted distribution with weight $[h(x)]^{-1}$ where h(.) is the failure rate. Let G(.) denote the survival function of Y. The relationship of the characteristics of equilibrium distribution with that of the parent distribution in the context of reliability are studied by Gupta (1984), Gupta and Kirmani (1990) and Hitha and Nair (1989). Some of the important identities among them are

(i) $G(x) = \dfrac{1}{\mu}\displaystyle\int_x^{\infty} S(t)\,dt$

(ii) $h_y(x) = \dfrac{1}{r(x)}$, where $h_y(x)$ is the failure rate of Y.

In the point of view of Deshpande et.al. (1986) the life distribution of a unit which ages more rapidly will come off worse in a comparison of S(x) and G(x). The wide spread applicability of weighted distribution in univariate case has prompted many researchers to extend the concept to the higher dimensions. However the applications to real problems in such cases have rarely been pointed out. If (X_1,X_2) be a random vector in the support of $\{(x_1,x_2) : 0 < x_1,x_2 < \infty\}$ with an absolutely continuous distribution function $F(x_1,x_2)$ or the survival function $S(x_1,x_2)$. Defining $w(x_1,x_2)$ be a non-negative weighted function with $E[w(x_1,x_2)] < \infty$, Mahfoud and Patil (1982) defined a bivariate weighted distribution as the distribution of the vector (Y_1,Y_2) with p.d.f.

$$s(x_1,x_2) = \frac{w(x_1,x_2)\, f(x_1,x_2)}{E[w(X_1,X_2)]} \tag{2}$$

when $w(x_1,x_2) = 1/h(x_1,x_2) = \dfrac{S(x_1,x_2)}{f(x_1,x_2)}$

$$E[w(x_1,x_2)] = \int_0^\infty\int_0^\infty \frac{S(x_1,x_2)}{f(x_1,x_2)} f(x_1,x_2)\,dx_1 dx_2$$

That is $E[w(x_1,x_2)] = E(x_1,x_2) = \mu$

$$\text{Hence}\quad g(x_1,x_2) = \frac{S(x_1,x_2)}{E(x_1,x_2)} = \frac{S(x_1,x_2)}{\mu} \tag{3}$$

Let G(.,.) denote the survival function of (Y_1, Y_2). Then

$$G(x_1,x_2) = \int_{x_1}^\infty\int_{x_2}^\infty \frac{S(t_1,t_2)}{\mu} dt_1 dt_2 \tag{4}$$

We can see that

$$h_y(x_1,x_2) = 1/r(x_1,x_2) \tag{5}$$

2.1. Reliability Concepts for Continuous Lifetime Distributions :

If X be a non-negative random variable representing lifetime of a system or a device having absolutely continuous distribution function

$F(x) = P(X \le x),\ x>0$

Then the survival function of X is denoted by S(x) and is defined as

$$S(x) = P(X > x), \\ = 1 - F(x),\quad x>0. \tag{6}$$

S(x) is a non-increasing continuous function with

$\lim_{x\to 0} S(x) = 1$ and $\lim_{x\to\infty} S(x) = 0.$

2.2. Hazard rate :

An important function that characterizes lifetime distribution is the hazard rate. It is denoted by h(x) and is defined as

$$h(x) = \lim_{\Delta x \to 0} \frac{P(x \le X < x + \Delta x / X > x)}{\Delta x}, \quad x > 0.$$

The hazard rate specifies the instantaneous rate of failure of a device in the next small interval of time Δx, given that the device has survived up to time x. Thus h(x)Δ x is the approximate probability of failure in the interval $[x, x+\Delta x)$, given survival up to time x. The hazard rate is also known as conditional failure rate in reliability and the age-specific failure rate in epidemiology. When X is absolutely continuous, the hazard rate is expressed as

$$h(x) = \frac{f(x)}{S(x)} = \% \frac{d}{dx} \log S(x) \tag{7}$$

Integrating equation (7) with respect to x, we obtain

$$S(x) = \exp\left[-\int_0^x h(u)du,\right] \tag{8}$$

Which shows that h(x) characterizes the distribution of X. The pdf of X can also be represented as

$$f(x) = h(x) \exp\left[-\int_0^x h(u)du,\right] \tag{9}$$

2.3. Mean Residual Life Function :

Mean residual life function plays an important role in reliability, survival analysis and various other areas. It is often referred as life expectancy or expectation of life in demography. The mean residual life function (mrl) of X, m(x), is defined as the mean of the residual life (X-x/X>x), More explicitly,

m(x) = E(X-x/X>x),

$$= \frac{1}{S(x)} \int_x^\infty (u - x) f(u)du,$$

$$= \frac{1}{S(x)} \int_x^\infty S(u)du, \tag{10}$$

2.4. Variance Residual Life Function:

Another function which has also generated interest in the recent years is the variance residual life function. It is denoted by $\sigma^2(x)$ and is defined as

$$\sigma^2(x) = E\left[(X - x)^2 / X > x\right] - m^2(x)$$

$$= \frac{1}{S(x)} \int_x^\infty (u - x)^2 f(u)du - m^2(x),$$

$$= \frac{2}{S(x)} \int_x^\infty (u - x)S(u)du - m^2(x),$$

$$= \frac{2}{S(x)}\int_x^{\infty}\int_t^{\infty} S(u)dudt - m^2(x), \qquad (11)$$

obtained by integrating by parts on each of the steps. Abouammoh *et.al*. (1990) showed that the variance residual life together with mean residual life function characterizes the distribution of X through the identity

$$S(x) = \exp\left[-\int_0^x \frac{\frac{d}{du}\sigma^2(u)}{\sigma^2(u)-m^2(u)}du\right] \qquad (12)$$

3.0. Setting the Clock Back to Zero Property :

Rao and Talwalker (1990) introduced this concept of setting the clock back to zero (SCBZ) property. A family of life distributions $\{f(x,\theta), x \geq 0, \theta \in \Theta\}$ is said to have the SCBZ property if the form of $f(x,\theta)$ remains unchanged under the following three operations, except for the value of parameters, that is

$$f(x,\theta) \rightarrow f(x,\theta^*) \quad \text{where } \theta^* \in \Theta \qquad (13)$$

(i). Truncating the original distribution at some point $x_0 \geq 0$.

(ii). Considering the observable distribution for life time $X \geq x_0$ and

(iii). Changing the origin by means of the transformation given by $X_1 = X - x_0$, so that $X_1 \geq 0$.

In terms of the survival function $S(x,\theta)$, the definition can be restated as the following. A family of life distributions $\{S(x,\theta), x \geq 0, \theta \in \Theta\}$ is said to have the SCBZ property if for each $x_0 \geq 0$ and $\theta \in \Theta$, the survival function satisfies the equation

$$S(x+x_0, \theta) = S(x_0, \theta)\, S(x, \theta^*) \qquad (14)$$

with $\theta^* = \theta^*(x_0) \in \Theta$

The random variable X is said to have SCBZ property if

$$P(X \geq x+x_0 / X \geq x_0) = P(X^* \geq x), \quad \ldots \quad (15)$$

For Exponential, Pareto type II, finite range, Gompertz, linear hazard model, the model for time to tumor has been given by Rao (1990) possess this property. Rao *et.al.* (1993b) extended the notation of SCBZ in the bivariate case which he called the extended SCBZ property. Consider an individual exposed simultaneously to two risks R_1 and R_2 with hypothetical life times X_1 and X_2 respectively. The joint survival function of X_1 and X_2 is defined by $S(x_1,x_2,\theta)$, $0 \leq x_1,x_2 < \infty$, where θ is the parameter or a vector of parameters. The survival function of the individual up to age x_0 can have the idea that the individual's hypothetical life times satisfy $X_1 \geq x_0$ and $X_2 \geq x_0$. The conditional distribution of the additional survival time of an individual due to risk R_1 given that the individual has survived for a time to x_0 units is

$$P(X_1 \geq x_1+x_0 \;/\; X_1 \geq x_0,\; X_2 \geq x_0) = \frac{S(x_1+x_0, x_0, \theta)}{S(x_0, x_0, \theta)} \qquad (16)$$

In a similar way,

$$P(X_2 \geq x_2+x_0 \;/\; X_1 \geq e\text{''}x_0,\; X_2 \geq x_0) = \frac{S(x_1, x_2+x_0, \theta)}{S(x_0, x_0, \theta)} \qquad (17)$$

Using this notations Rao *et.al*. (1993b) defined SCBZ property in the bivariate case as follows.

A class of bivariate life distributions $\frac{S(x_1+x_0,x_0,\theta)}{S(x_0,x_0,\theta)} = S(x_1,x_0,\theta^*)$

and $$\frac{S(x_1,x_2+x_0,\theta)}{S(x_0,x_0,\theta)} = S(x_0,x_2,\theta^{**})$$

Where $\theta^* = \theta^*(x_0)$ and $\theta^{**} = \theta^{**}(x_0) \in \Theta_0$ where Θ_0 denote the boundary of Θ.

They have showed that the life expectancy vector $(r_1(x_0,x_0,\theta), r_2(x_0,x_0,\theta))$ has a closed form, since

$$r_1(x_0,x_0,\theta) = E_\theta\left(X_1 - x_0 / X_1 \geq x_0, X_2 \geq x_0\right)$$

$$= \frac{1}{S(x_0,x_0,\theta)}\int_0^\infty\int_0^\infty (x_1-x_0) f(x_1,x_2,\theta) dx_1 dx_2,$$

where $f(x_1,x_2,\theta)$ is the joint p.d.f. of (X_1,X_2). That is

$$r_1(x_0,x_0,\theta) = \int_0^\infty \frac{S(x_1+x_0,x_0,\theta)}{S(x_0,x_0,\theta)} dx_1$$

$$= \int_0^\infty S(x_1,x_0,\theta^*) dx_1$$ Similarly $r_2(x_0,x_0,\theta) = \int_0^\infty S(x_0,x_2,\theta^{**}) dx_2$.

The examples cited in Rao *et.al*. (1993b) include the bivariate exponential distributions proposed by Marshall-Olkin (1967) and Gumbel (1960), Bivariate Gompertz and Bivariate Pareto models.

3.1. Setting the Clock Back to Zero Property in Equilibrium Distribution :

If X be a random variable admitting an absolutely continuous survival function $S(x,\infty)$ with respect to the Lebesgue measure in the support of the set of non-negative real with a finite mean μ. Then a new random variable Y with p.d.f.

$$G(x,\theta) = \frac{S(x,\theta)}{\mu}, x > 0$$

is said to be the random variable corresponding to the equilibrium distribution.. A family of survival distributions $\{S(x_1,x_2,\theta): x_1,x_2>0, \theta \in \Theta\}$ is said to have conditional SCBZ(1) property if it satisfies the equations

$$\left.\begin{array}{l} \frac{G_1(t_1+s_1,t_2,\theta)}{G_1(t_1,t_2,\theta)} = G_1(s_1,t_2,\theta^*) \\ \\ \textit{and} \\ \\ \frac{G_2(t_1,t_2+s_2,\theta)}{G_2(t_1,t_2,\theta)} = G_2(t_1,s_2,\theta^*) \end{array}\right\} \quad (18)$$

for all $t_1, t_2, s_1, s_2 > 0$ where θ^* and θ^{**} belong to Θ.

$$G_1(t_1,t_2,\theta) = P(X_1>t_1/X_2>t_2,\theta)$$

$$= \frac{P(X_1 > t_1, X_2 > t_2, \theta)}{P(X_2 > t_2, \theta)}$$

$$G_1(t_1,t_2,\theta) = \frac{S(t_1,t_2,\theta)}{S(0,t_2,\theta)} \quad \text{and} \quad G_2(t_1,t_2,\theta) = \frac{S(t_1,t_2,\theta)}{S(t_1,0,\theta)}$$

3.2. Conditional Setting the Clock Back to Zero (2) Property :

In accordance with the conditional lack of memory property defined by Nair and Nair (1991), a new approach can be taken to define the SCBZ property in the bivariate case. We call that property as conditional SCBZ(2) property. If (X_1,X_2) be a non-negative random vector defined on R_2^+ with an absolutely continuous survival distribution $R(.,.,\theta)$. Denote the conditional survival function of X_i given $X_j=t_j$ by $S_i(t_i,t_j,\theta)$ for all $i,j=1,2;\ i \neq j$.

That is $S_i(t_i,t_j,\theta) = P(X_i \geq t_i / t_i \leq X_j \leq t_j+dt_j)$, $i,j = 1,2;\ i \neq j$.

where dt_j is a small increment in t_j.

3.3. SCBZ properties in Bivariate Equilibrium Distribution :

The bivariate equilibrium distribution has a density of the form

$g(x_1,x_2,\theta) = S(x_1,x_2,\theta)/\mu,\quad x_1,x_2>0$

where $S(x_1,x_2,\theta)$ is the survival function of the parent distribution. The case of univariate situation of $h_y(x_1,x_2,\theta)$ denote the scalar failure rate of (Y_1,Y_2) and $r(x_1,x_2,\theta)$ denote the scalar mean residual life of (X_1,X_2), we will get the identity , which is

$h_y(x_1,x_2,\theta) = 1/ r(x_1,x_2,\theta)$.

3.4. Multivariate Setting the Clock Back to Zero Properties :

The SCBZ properties in the bivariate case can also be extended to more than two variables cases. A class of multivariate life distributions $\{R(x_1,x_2,\ldots, x_n,\theta), x_i \geq 0, \theta \in \Theta\}$ is said to have multivariate SCBZ(1) property if for each $\theta \in \Theta$ and $x_0 \geq 0$, the survival function satisfies the condition

$$\frac{S(x_1+x_0, x_2+x_0,\ldots, x_n+x_0,\theta)}{S(x_0,x_0,\ldots,x_0,\theta)} = S(x_1,x_2,\ldots,x_n,\theta^*) \tag{19}$$

where = $\theta^* = \theta^*(x_0) \in \Theta$.

3.4. SCBZ Property of Discrete Distributions :

The study of this property in the discrete set up is of more interest, since in actual practice the life of the components are measured in discrete time units, that is the time is measured discrete time units, that is the time is measured discretely as the completed years of life or as number of cycles. The difficulties, in measuring the time continuously are discussed by authors like Xekalaki (1983), Cox (1972) and Kalbfleish and Prentice (1980).

$k(x) = \dfrac{f(x)}{S(x)}$, the vector valued failure rate of Johnson and Kotz (1975) (20)

$h(x) = -\nabla \log S(x)$, Where $\nabla = \left(\dfrac{\partial}{\partial x_1}, \dfrac{\partial}{\partial x_2}, ..., \dfrac{\partial}{\partial x_p}\right)$ is the p-dimensional gradient operator, so that if $h_i(x)$ is the i^{th} component of h(x)

$$h_i(x) = -\frac{\partial \log S(x)}{\partial x_i}, i = 1,2,..., p. \tag{21}$$

Further, the mean residual life function of X is Zahedi (1985)

m(x) = E [X-x/X > x] with i^{th} component

$m_i(x) = E[X_i - x_i/X > x]$,

$$= \frac{1}{S(x)} \int_{x_1}^{\infty} S(x_i, t_i)\, dt_i \tag{22}$$

where $(x_{(i)}, t_i)$ stands for the vector x in which the i^{th} element x_i is replaced by t_i. differentiating equation (23) partially with respect to x_i,

$$\frac{\partial m_i(x)}{\partial x_i} = \frac{1}{S^2(x)}\left[-S^2(x) - \frac{\partial S(x)}{\partial x_i}\int_{x_i}^{\infty} S(x_{(i)}.t_i)dt_i\right], = -1 + h_i(x)\, m_i(x),$$

We get the relationship between equation (21) and equation (22) as,

$$h(x_i) = \frac{1}{m_i(x)}\left(1 + \frac{\partial m_i(x)}{\partial x_i}\right), i = 1,2,..., p.$$

We also need the concept of product moment of residual life defined as Nair *et. al.* (2004)

$$P_0(x) = E\left[\prod_{i=1}^{p}(X_i - x_i)\middle| X > x\right], = \frac{1}{S(x)}\int_{(x,\infty)} S(t)dt, \tag{23}$$

The multivariate equilibrium distribution of order n based on S(x) is defined recursively through the relations

$$S_n(x) = \frac{\int_{(x,\infty)} S_{n-1}(t)\,dt}{\int_{(0,\infty),} S_{n-1}(t)\,dt}, n = 1,2,....$$

with $S_0(x) = S(x)$ and

$\mu_n = \int_{(0,\infty)} S_n(t)\,dt$.

Some interpretations offered to the univariate equilibrium distributions can be extended to the MVED's as well.

$$f_w(x) = \frac{[k_{n-1}(x)]^{-1} f_{n-1}(x)}{E[(k_{n-1}(x))^{-1}]},$$

$$= S_{n-1}(x)\left[\int_{(0,\infty)} \frac{S_{n-1}(x)}{f_{n-1}(x)} f_{n-1}(x)dx\right]^{-1},$$

$$= S_{n-1}(x)\left[\int_{(0,\infty)} S_{n-1}(x)dx\right]^{-1}.$$

The last expression is the density function corresponding to Sn(x). Secondly, in the univariate case, if X is a non-negative random variable with density function f(x), then the distribution of W Z is the equilibrium distribution,

$f_W(x) = \frac{xf(x)}{E(X)}$ and Z is uniform over (0 ; 1) independently of W .

The n-th order equilibrium distribution of the mixed p.f.

$p(x) = \int_0^\infty p(x/\theta)dU(\theta),$ is given by

$p_n(x) = \int_0^\infty p_n(x/\theta)dU_n(\theta),$

where $dU_n(\theta) = \frac{E[X^{(n)}/\theta]dU(\theta)}{E[X^{(n)}]}.$

Based on the 1^{st} order and n^{th} order equilibrium distribution, their estimates, K-S distance and the corresponding p-values are presented in the table along with m(x) and $\sigma^2(x)$.

In manpower planning the concept of SCBZ is used to derive the Expected time to recruitment and its variance by several researchers. In the case of Equilibrium Distribution is not possible derive the mean and variance where as the 1^{st} order and n^{th} order statistics derived for the specific distribution and it is attempted by many researchers. Hence in this content it would be difficult task to derive the same and it is one of the important problem for further research.

Table 1. The K-S Distance based on 1^{st} order and n^{th} order Equilibrium Distribution

Parameters	m(x)	$\sigma^2(x)$	S(x)	K-S distance	p-value
θ_1	1.056	32.987	38.956	0.0438	0.9147
θ_2	1.190	45.987	47.432	0.0689	0.8023
θ_3	1.334	61.908	65.843	0.0850	0.9137
θ_4	1.509	74.895	78.902	0.0895	0.9768
θ_5	1.734	82.675	89.369	0.0925	0.8989
θ_6	1.897	97.412	102.784	0.0986	0.9608

5.0. Conclusion

The conceptualizations of mathematical and Stochastic model help the process of finding the optimal solutions and also the implementation of the same. However it is very important to identify the appropriate probability distribution that would portray the realities. The identification appropriate distribution is an important step. To make a brief introduction on equilibrium distributions and their applications, which explain the motivation and objectives of the present study. Some basic concepts in reliability that help to explain the existing results. We considered equilibrium distributions of non-negative continuous random variables representing lifetimes of components or devices. The survival function of equilibrium distribution of order n and that of the baseline distribution were linked through the moments of the residual lives of the original distribution. Further, the hazard rate and the mean residual life function of the higher order equilibrium distribution were expressed in terms of their respective lower orders. We also derived the identity connecting the mean residual life functions of the original distribution, the equilibrium distribution of order n and the residual life distribution of the equilibrium renewal process. It can be proved that the SCBZ property preserves in the equilibrium distributions of univariate continuous and discrete cases. The bivariate SCBZ (2) property preserves in the equilibrium distribution in continuous distributions and vice versa. The results show that the p value is significant and gives better results for the 1^{st} order and n^{th} order Equilibrium Distribution when compared with m(x) and $\sigma^2(x)$. The validity of the numerical results gives only the best estimates when the real data is collected from the industry.

6.0. Scope of Future Research :

For further investigation, these are many areas of an organization or industry in which the optimal recruitment policies for human resource planning is quite necessary. It would be very much useful in every sector of human activity. First of all it is imperative to identify those areas of human activity, policies relating to the demand for manpower and supply are essential, and are usually at disequilibrium. Especially in the area of specialist skill, it becomes necessary to identify where the disequilibrium exists and also were there is interruption in the work schedule due to shortage of manpower. The identification of such areas, the type of problems involved and the conversion of a real life situation into a mathematical model are essential to develop human resource management, which will yield profits not only to the management but also to the society itself. The conceptualization of different recruit policies developed in this paper help the process of finding the optimal solutions and also the implementation of the same. However it is very important to identify the appropriate equilibrium distribution that would portray the realities and real life data is an important procedure. Once this is taken care of then the models can be used in solving real life problems.

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