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On Symmetric Circulant Fuzzy Matrices

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Abstract

*T*he basic concepts, theorems and properties of circulant fuzzy matrix are introduced also we discussed k- symmetric circulant fuzzy and s- symmetric circulant fuzzy matrices with examples.

Key words : Circulant Fuzzy matrix, Symmetric Circulant Fuzzy matrix, s- Symmetric Circulant Fuzzy matrix, k- Symmetric Circulant Fuzzy matrix.

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Introduction

A square matrix A is Symmetric if $A=A^T$ and k-symmetric if $A=KAK^T$ [2,3,4,5,7] Any matrix with entries in [0,1] and matrix operator defined by fuzzy logical operators are called fuzzy matrix[1,8]. Symmetric, k-symmetric circulant and s-Symmetric circulant are defined in the year 2016 [9]. Throughout in this paper all matrices considered over a fuzzy algebra F with support [0, 1] under max- min operations for $a, b \in F$, $a+ b = \max \{a, b\}$, $a. b = \min\{a, b\}$. The concept of s- symmetric matrices, k- symmetric matrices was introduced in[1,4]. In this paper our intension k- symmetric circulant fuzzy and s- symmetric circulant matrices are discussed.

Preliminaries and Notations :

For any $A \in F^{n\times n}$ is a fuzzy circulant matrix then A^T, A^S are transpose, secondary transpose respectively. Let k be a fixed product of disjoint transposition in S_n and K be the permutation matrix associated with k, V is a permutation matrix with units in the secondary diagonal. Clearly $K^2 = I$, $K^T = K$, $V^2 = I$, $V^T = V$.

Definition 1.1 Circulant Fuzzy Matrix (Cfm) :

For any given $a_0, a_1, \ldots, a_{n-1} \in F^{n \times n}$ be the Fuzzy circulant matrix $A = (a_{ij})_{n \times n}$ is defined by $(A_{ij}) = (a_{j-1(\text{mod } n)})$

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-1} \\ a_{n-1} & a_0 & a_1 & \cdots & a_{n-2} \\ a_{n-2} & a_{n-1} & a_0 & \cdots & a_{n-3} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_1 & a_2 & a_3 & \cdots & a_0 \end{bmatrix}$$

Definition 1.2 Symmetric Circulant Fuzzy Matrix (SCFM) :

A matrix $A \in F^{n \times n}$ is said to be symmetric circulant fuzzy matrix if $A = A^T$

Example 2.1

$$A = \begin{bmatrix} 0.1 & 0.3 & 0.3 \\ 0.3 & 0.1 & 0.3 \\ 0.3 & 0.3 & 0.1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0.1 & 0.3 & 0.3 \\ 0.3 & 0.1 & 0.3 \\ 0.3 & 0.3 & 0.1 \end{bmatrix}$$

Definition 1.3 k – Symmetric Circulant fuzzy matrix (k-SCFM) :

A matrix $A \in F^{n \times n}$ is said to be k - symmetric circulant fuzzy matrix if $A = KA^TK$ where K be the permutation associated with k.

Example 2.2

$$A = \begin{bmatrix} 0.3 & 0.5 & 0.5 \\ 0.5 & 0.3 & 0.5 \\ 0.5 & 0.5 & 0.3 \end{bmatrix} \qquad KA^TK = \begin{bmatrix} 0.3 & 0.5 & 0.5 \\ 0.5 & 0.3 & 0.5 \\ 0.5 & 0.5 & 0.3 \end{bmatrix} \qquad \therefore A = KA^TK$$

Theorem 3.1

Let $A \in F^{n \times n}$ is said to be k - symmetric circulant fuzzy matrix then $A = KA^TK$

Proof: A matrix $A \in F^{n \times n}$ is said to be k -symmetric circulant fuzzy matrix if $A = KA^TK$ To prove A^T is said to be k - symmetric circulant fuzzy matrix.

$$A^T = (KA^TK)^T$$
$$= K^T(A^T)^TK^T$$
$$A^T = KAK$$

Theorem 3.2

Let $A \in F^{n \times n}$ be k - symmetric circulant fuzzy matrix if A^{-1} is circulant fuzzy matrix then A^{-1} is also k - symmetric circulant fuzzy matrix.

Proof : A matrix $A \in F^{n \times n}$ is said to be k -symmetric circulant fuzzy matrix if $A = KA^TK$ To Prove A^{-1} is k - symmetric circulant fuzzy matrix.

$$A^{-1} = (KA^TK)^{-1}$$
$$= K^{-1}(A^T)^{-1}K^{-1}$$
$$= K(A^T)^{-1}K$$
$$A^{-1} = K(A^{-1})^TK$$

Theorem 3.3

If A and B are k - symmetric circulant fuzzy matrix then A + B is also k-symmetric circulant fuzzy matrix.

Proof: Since A and B are k - symmetric circulant fuzzy matrix then

$A = KA^TK$ and $B = KB^TK$

To prove A+B is k - symmetric circulant fuzzy matrix.

$$A+B = KA^TK + KB^TK$$
$$= K [A^T+B^T] K$$
$$A+B = K [A+B]^T K$$

Theorem 3.4

If A and B are k - symmetric circulant fuzzy matrix then AB is also k - symmetric circulant fuzzy matrix.

Proof: Since A and B are k - symmetric circulant fuzzy matrix then

$A = KA^TK$ and $B = KB^TK$

To prove AB is k - symmetric circulant fuzzy matrix.

$$AB = (KA^TK) (KB^TK)$$
$$= (KA^T) K^2(B^TK)$$
$$= KA^TB^TK$$
$$= K (BA)^T K$$
$$AB = K (AB)^T K$$

Theorem 3.5

Let $A \in F^{n \times n}$ be k - symmetric circulant fuzzy matrix and K is the permutation matrix then KA is also k - symmetric circulant fuzzy matrix.

Proof: Since A and A^T are k - symmetric circulant fuzzy matrices then

$A = KA^TK$, $A^T = KAK$

To Prove KA is k - symmetric circulant fuzzy matrix.

$$KA = K(KA^TK)$$
$$= KK^TA^TK$$
$$= K (AK)^TK$$
$$KA =K (KA)^TK$$

Theorem 3.6

Let $A \in F^{n \times n}$ be k - symmetric circulant fuzzy matrix and K is the permutation matrix then AK is also k - Symmetric circulant fuzzy matrix.

Proof: Since A and A^T are k - symmetric circulant fuzzy matrices then

$A = KA^TK$, $A^T = KAK$

To Prove AK is k - symmetric circulant fuzzy matrix.

$$AK = (KA^TK)K$$
$$= KA^TK^TK$$
$$= K (KA)^TK$$
$$AK =K (AK)^TK$$

Theorem 3.7

If $A \in F^{n \times n}$ be k - symmetric circulant fuzzy matrix then AA^T and A^TA is also k - symmetric circulant fuzzy matrix.

Proof: Since A and A^T are k - symmetric circulant fuzzy matrices then

$A = KA^TK$, $A^T = KAK$.

To Prove AA^T is k - symmetric circulant fuzzy matrix

$AA^T=(KA^TK)\ (KAK)$

$= K\ (A^TA)K$

$AA^T = K\ (AA^T)^TK$

Similarly We will prove A^TA is also k - symmetric circulant fuzzy matrix.

Theorem 3.8

If $A \in F^{n\times n}$ be k - symmetric circulant fuzzy matrix then $A+A^T$ is also k - symmetric circulant fuzzy matrix.

Proof: Since A and A^T are k - symmetric circulant fuzzy matrices then

$A = KA^TK$, $A^T = KAK$.

To Prove $A +A^T$ is k - symmetric circulant fuzzy matrix

$A+A^T= KA^TK + KAK$

$= K\ (A^T + A)\ K$

$A+A^T = K\ (A +A^T)^TK$

Theorem 3.9

If A and B are k-symmetric circulant fuzzy matrices then AB+BA is also k-symmetric circulant fuzzy matrix

Proof: Given A and B are k - symmetric circulant fuzzy matrices then

$A = KA^TK$ and $B = KB^TK$

To prove AB+BA is k - symmetric circulant fuzzy matrix.

$AB + BA = (KA^TK)(\ KB^TK) + (KB^TK)(KA^TK)$

$= (KA^T)K^2(B^TK)+(KB^T)K^2(A^TB)$

$= K\ A^TB^T\ K + K\ B^TA^T\ K$

$= K\ (BA)^T\ K + K\ (AB)^T\ K$

$= K\ (AB+\ BA^T\ K$

Theorem 3.10

Let A is k-symmetric circulant fuzzy matrix and if α , $\beta \geq 0$ are scalar Fuzzy and $\alpha+\beta= 1$ then $\alpha A+\beta B$ is also k-symmetric circulant fuzzy matrix.

Proof: Since A and B are k-symmetric circulant fuzzy matrices then

$A = KA^TK$ and $B = KB^TK$

To prove $\alpha A+\beta B = \alpha\ KA^TK + \beta\ KB^TK$

$= K\ [\alpha\ A^T + \beta\ B^T]\ K$

$= K\ [\alpha\ A + \beta\ B]^T\ K$

Remarks: If α , $\beta \geq 0$ and $\alpha+\beta= 1$ with $\alpha A+\beta B$ are symmetric circulant fuzzy matrix so the set of all symmetric circulant fuzzy matrices over the permutation k is a convex set.

Theorem 3.11

If A_1 and A_2 are k-symmetric circulant fuzzy matrices then $(A_1{}^TA_2A_1)$ and $(A_2{}^TA_1A_2)$ are also k-symmetric circulant fuzzy matrix.

Proof: Since A_1 and A_2 are k-symmetric circulant fuzzy matrices then

$A_1 = KA_1{}^TK$ and $A_2 = KA_2{}^TK$. Since $A_1{}^T$ and $A_2{}^T$are also k-symmetric circulant fuzzy matrices then $A_1{}^T = KA_1K$

and $A_2^T = KA_2K$

To prove $A_1^TA_2A_1$ is k-symmetric circulant fuzzy matrix.

$$A_1^TA_2A_1 = (KA_1K)(KA_2^TK)(KA_1^TK)$$
$$= KA_1K^2A_2^TK^2A_1^TK$$
$$= K(A_1A_2^TA_1^T)K$$
$$= K(A_1^TA_2A_1)^TK$$

Similarly We will prove $A_2^TA_1A_2$ is also k-symmetric circulant fuzzy matrix.

Definition 1.3 s– Symmetric Circulant fuzzy matrix (s-SCFM) :

A matrix $A \in F^{n \times n}$ be s - symmetric circulant fuzzy matrix if $A=A^S=VA^TV$ where V is a permutation matrix with units in the secondary diagonal.

Example 2.3

$$A=\begin{bmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{bmatrix},\ V=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\quad A^T=\begin{bmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{bmatrix},\ A^S=\begin{bmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{bmatrix},\ VA^TV=\begin{bmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{bmatrix}$$

Theorem 3.12

A matrix $A \in F^{n \times n}$ be s - symmetric circulant fuzzy matrix then A^T is also s - symmetric circulant fuzzy matrix.

Proof: Since A is s - symmetric circulant fuzzy matrix then $A=A^S=VA^TV$

To prove A^T is s - symmetric circulant fuzzy matrix.

$$A = A^S$$
$$A^T = (VA^TV)^T$$
$$= V(A^T)^TV$$
$$= VAV$$
$$A^T = VA^SV$$

Theorem 3.13

If A and B are s - symmetric circulant fuzzy matrices then (A+B) is also s - symmetric circulant fuzzy matrix.

Proof: Since A and B are s - symmetric circulant fuzzy matrices then

$A=A^S=VA^TV$ and $B=B^S=VB^TV$

To prove (A+B) is s - symmetric circulant fuzzy matrix.

$$(A+B)^S = (VA^TV + VB^TV)^S$$
$$= V(A^T)^SV + V(B^T)^SV$$
$$= V[(A^S)^T + (B^S)^T]V$$
$$= V[A^T+B^T]V$$
$$(A+B)^S = V[A+B]^TV = A+B$$

Theorem 3.14

If A and B are s - symmetric circulant fuzzy matrices then (AB) is also s - symmetric circulant fuzzy matrix.

Proof: Since A and B are s - symmetric circulant fuzzy matrices then

$A=A^S=VA^TV$ and $B=B^S=VB^TV$.

To prove (AB) is s - symmetric circulant fuzzy matrix.

$$(AB)^S = (VA^TV\,VB^TV)^S$$
$$= [VA^TV^2B^TV]^S$$
$$= [VA^TB^TV]^S$$
$$= [V(AB)^TV]^S$$
$$= V\,[(AB)^T]^S\,V$$
$$= V\,[(AB)^S]^T\,V$$
$$= V\,(AB)^T\,V$$

Theorem 3.15

A matrix $A \in F^{n\times n}$ be s - symmetric circulant fuzzy matrix and V is a permutation matrix with units in the secondary diagonal then VA and AV also
s - symmetric circulant fuzzy matrix.

Proof: Since A is s - symmetric circulant fuzzy matrix then $A=A^S=VA^TV$

To prove VA is s - symmetric circulant fuzzy matrix.

$$VA = VA^S$$
$$= V\,(VA^TV)$$
$$= V\,A^T\,V\,V$$
$$= V\,A^T\,V^T\,V$$
$$= V\,(VA)^T\,V$$

Similarly we will prove AV is s - symmetric circulant fuzzy matrix.

Theorem 3.16

If $A \in F^{n\times n}$ be s - symmetric circulant fuzzy matrix then AA^T and A^TA are also s - symmetric circulant fuzzy matrix.

Proof: A matrix A is said to be s - symmetric circulant fuzzy matrix then $A=A^S=VA^TV$.

Since A^T is s - symmetric circulant fuzzy matrix then $A=A^T=VA^SV$

To prove AA^T is s - symmetric circulant fuzzy matrix

$$AA^T = (AA^T)^S$$
$$= (A^T)^S\,A^S$$
$$= (A^S)^T\,A^S$$
$$= (VA^TV)^T\,(VA^TV)$$
$$= (V\,(A^T)^TV)(VA^TV)$$
$$= V\,AA^T\,V$$
$$= V\,(A^TA)^T\,VA$$
$$AA^T=(AA^T)^S = V\,(AA^T)^T\,V$$

Similarly we will prove $A^TA = V\,(A^TA)^TV$

Theorem 3.17

If $A \in F^{n\times n}$ be s - symmetric circulant fuzzy matrix then $A+A^T$ is also
s - symmetric circulant fuzzy matrix.

Proof: A matrix A is said to be s - symmetric circulant fuzzy matrix then $A=A^S=VA^TV$. To Prove $A + A^T = (A+A^T)^S$

$$= A^S + (A^S)^T$$
$$= VA^TV + (VA^TV)^T$$
$$= VA^TV + V(A^T)^TV$$
$$= VA^TV + VAV$$
$$= V[A^T + A]V$$
$$= V[A+A^T]V$$

Theorem 3.18

If A and B are s - symmetric circulant fuzzy matrix then AB + BA is also s - symmetric circulant fuzzy matrix.

Proof: A matrix A is said to be s - symmetric circulant fuzzy matrix then $A=A^S=VA^TV$.

To prove $(AB+BA)^S = [(VA^TV)(VB^TV)+(VB^TV)(VA^TV)]^S$
$$= [V(A^TB^T)V+V(B^TA^T)V]^S$$
$$= [V(BA)^TV+V(AB)^TV]^S$$
$$= [V(BA)^TV]^S+[V(AB)^TV]^S$$
$$= V\{[(BA)^S]^T + [(AB)^S]^T\}V$$
$$= V[(AB)^T+(BA)^T]V$$
$$= V(AB+BA)^T V$$

Theorem 3.19

Let A is s-symmetric circulant fuzzy matrix and if $\alpha, \beta \geq 0$ are scalar Fuzzy and $\alpha+\beta= 1$ then $\alpha A+\beta B$ is also s-symmetric circulant fuzzy matrix.

Proof: Since A and B are s-symmetric circulant fuzzy matrices then

$$A=A^S= VA^TV \text{ and } B=B^S= VB^TV$$

To prove $(\alpha A+ \beta B)^S = \alpha (VA^TV) + \beta (VB^TV)$
$$= V[\alpha A^T + \beta B^T] V$$
$$= V[\alpha A + \beta B]^T$$

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