

## On Induced Affine Connection of Almost Hyperbolic Hermitian Manifold

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### Abstract

The purpose of the present paper is to study properties of hyperdistribution of real hypersurfaces of almost hyperbolic Hermitian manifold with induced affine connection.

*Key words:* Almost Hermitian manifolds, Real submanifolds, induced affine connection.

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### 1. Introduction

*Definition:* Let us consider a differentiable manifold  $M$  of class  $C^\infty$  endowed with a tensor field  $F$  of type  $(1, 1)$  such that

$$\tilde{F}^2 = I, \text{ i.e. } \dots \text{ or } \tilde{F}_k^h \tilde{F}_i^k = \delta_i^h,$$

and  $g(FX, FY) + g(X, Y) = 0$

Then we say that  $g$  is compatible with structure  $F$  and  $(F, g)$  is called almost hyperbolic Hermitian structure and the manifold  $M$  with this structure is called almost hyperbolic Hermitian manifold.

**Summation Convention:** In the sequel, manifolds, tensor fields, connections and mappings we consider are assumed to be differentiable and of class  $C^\infty$  unless otherwise stated and

the indices  $a, b, c, d, e, \dots$  run over the range  $\{1, 2, \dots, 2n + 1\}$ , the summation convention being used with respect to this system of indices.

Let there be given, on a manifold  $M$  of odd dimension  $2n + 1 (\geq 3)$ , a tensor field  $f$  of type  $(1, 1)$ , a vector field  $\xi$  and a 1-form  $\theta$  satisfying

$$(1.1) \quad f^2 = I - \theta \otimes \xi, \quad f(\xi) = 0, \quad \theta(f) = 0, \quad \theta(\xi) = 1,$$

$I$  being the identity tensor field of type  $(1, 1)$ , or

$$(1.2) \quad f_e^a f_b^e = \delta_b^a - \theta \otimes \xi, \quad f_e^a \xi^e = 0,$$

$$\theta_e f_b^e = 0, \quad \theta_e \xi^e = 1,$$

$f_b^a, \xi^a$  and  $\theta_b$  denoting components of  $f, \xi$  and  $\theta$  respectively. Then the triple  $(f, \xi, \theta)$  is called

an almost para contact structure in M.

We define tensor fields S of type (1, 2), G of type (0, 2), T of type (0, 2), P of type (1, 1) and Q of type (0, 1) as those with components

$$(1.3) S_{cb}^a = f_c^e \nabla_e f_b^a - f_b^e \nabla_e f_c^a - (\nabla_c f_b^e - \nabla_b f_c^e) f_e^a + (\nabla_c \theta_b - \nabla_b \theta_c) \xi^a,$$

$$(1.4) G_{cb} = f_c^e (\nabla_e \theta_b - \nabla_b \theta_e),$$

$$(1.5) T_{cb} = G_{cb} - G_{bc},$$

$$(1.6) P_b^a = -[\xi^e \nabla_e f_b^a - (\nabla_e \xi^a) f_b^e + (\nabla_b \xi^e) f_e^a],$$

$$(1.7) Q_b = -[\xi^e \nabla_e \theta_b + (\nabla_b \xi^e) \theta_e],$$

respectively, where  $\nabla$  denotes the operator of covariant differentiation with respect to an arbitrary symmetric affine connection in M. We easily see that these tensor fields are independent of the symmetric connection  $\nabla$  used to define them. Then S and G are respectively called the torsion tensor and the Levi tensor of  $(f, \xi, \theta)$ . The following propositions are well known<sup>4</sup>:

$$(A_1) S = 0 \text{ implies } T = 0, P = 0 \text{ and } Q = 0;$$

$$(A_2) P = 0 \text{ implies } Q = 0.$$

When the tensor field S vanishes identically, the almost para contact structure  $(f, \xi, \theta)$  is said to be normal.

We now state an elementary lemma for later use. Let V be a vector space over real number field with almost hyperbolic Hermitian structure F. That is, F:  $V \rightarrow V$  is a linear transformation satisfying  $F^2 = I$  and  $g(FX, FY) + g(X, Y) = 0$ . Then V is necessarily even-

dimensional, say  $\dim V = 2n + 2 (\geq 4)$ . Take arbitrarily a  $(2n + 1)$ -dimensional subspace W of V. Then FW is also  $(2n + 1)$ -dimensional. We can now state

*Lemma 1.1:* Put  $D = W \cap FW$  and  $N = D - FW$ . Then  $FD = D$ ,  $FN \subset W$ ,  $V = W + FW$ ,  $\dim D = 2n$ ,  $N = \{ax_0 + y / a \in \mathbb{R}, a \neq 0, y \in D\}$ ,  $x_0$  being a fixed element of N, and any element x of N is uniquely represented as  $x = ax_0 + y$  ( $a \in \mathbb{R}, y \in D$ ).

The subset N appearing in Lemma 1.1 has two connected components, each of which is homeomorphic to a Euclidean space of dimension  $2n + 1$ . The subset T is called the affine normal space to W in the vector space V with almost hyperbolic Hermitian structure F.

## 2. Hypersurfaces of almost hyperbolic Hermitian manifold:

Let M be a almost hyperbolic Hermitian manifold of real dimension  $2n + 2 (\geq 4)$  with almost hyperbolic Hermitian structure F, where F is a tensor field of type (1, 1) in M satisfying  $\tilde{F}^2 = I$ , i.e. . . .

$$(2.1a) \quad \tilde{F}_k^h \tilde{F}_i^k = \delta_i^h,$$

and

$$(2.1b) \quad g(FX, FY) + g(X, Y) = 0,$$

$\tilde{F}_i^h$  denoting components of  $\tilde{F}$ .

Let there be given a hypersurface M immersed in  $\tilde{M}$ . For each point P of M, denote the tangent space to  $\tilde{M}$  and that to M at P by  $T_P(\tilde{M})$  and  $T_P(M)$  respectively. Then the

subspace  $D_P = T_P(M) \cap \tilde{F} T_P(M)$  is  $2n$ -dimensional and hence the correspondence  $P \rightarrow D_P$  defines a distribution  $D$  of dimension  $2n$  in  $M$ . Since  $FD = D$ , we can define a tensor field  $J$  of type  $(1, 1)$  in  $D$  by  $JX = \tilde{F} X$ ,  $X$  being an arbitrary vector field belonging to  $D$ . Then  $\tilde{F}^2 = I$  implies  $J^2 = I_D$ , where  $I_D$  denotes the identity tensor field of type  $(1, 1)$  in  $D$ . Thus the  $D$  is called a hyperdistribution with almost hyperbolic Hermitian structure  $J$  in  $M$  and said to be induced in  $M$  from  $\tilde{F}$  by the immersion<sup>3</sup>.

Since the tangent space  $T_P(\tilde{M})$  is a vector space with almost hyperbolic Hermitian structure  $\tilde{F}$ , by Lemma 1.1 the subspace  $T_P(M)$  of  $T_P(\tilde{M})$  has its affine normal space  $N_P$ . We call  $N = \bigcup_{P \in M} N_P$  the affine normal bundle to the hypersurface  $M$ .

Since  $N_P$  has two connected components, each of which is homeomorphic to a Euclidean space,  $N$  has a global cross-section if  $M$  is orientable.

Let  $\bar{U}$  be a coordinate neighborhood of  $\tilde{M}$  such that any connected component  $U$  of  $\bar{U} \cap M$  is a coordinate neighborhood of  $M$ . In the sequel by  $U$  we mean such a coordinate neighborhood of  $M$ . Take a local cross-section  $C$  of the affine normal bundle  $N$  over  $U$  and call it a local affine normal to  $M$  in  $U$ . Then by Lemma 1.1  $\tilde{F} C$  is tangent to  $M$  in  $U$  and hence

$$(2.2) \quad \xi = \tilde{F} C$$

is a non-vanishing vector field in  $U$ . Next, for any vector field  $X$  in  $M$ , we can decompose  $\tilde{F} X$  uniquely as

$$(2.3) \quad \tilde{F} X = fX + \theta(X) C,$$

where  $fX$  is tangent to  $M$ . Thus  $f$  and  $\theta$  are a tensor field of type  $(1, 1)$  and a 1-form in  $U$  respectively. Applying  $\tilde{F}$  to (2.3) and using  $\tilde{F}^2 = I$ , we find  $X = (f^2 X + \theta(X)\xi) + \theta(fX) C$ , which implies

$$(2.4) \quad f^2 = I - \theta \otimes \xi, \quad \theta(f) = 0.$$

If we put  $X = \xi$  in (2.3), we obtain  $\tilde{F} \xi = f(\xi) + \theta(\xi) C$ . On the other hand (2.2) gives  $\tilde{F} \xi = C$ . Hence we get

$$(2.5) \quad f(\xi) = 0, \quad \theta(\xi) = 1.$$

Equations (2.4) and (2.5) show that the triple  $(f, \xi, \theta)$  is an almost contact structure in  $U$ , which is called an almost contact structure induced in  $M$  by an affine normal  $C$  in  $U$ . A vector field  $X$  in  $M$  belongs to  $D$  if and only if  $\tilde{F} X$  belongs to  $D$ . Thus, because of (2.3),  $X$  belongs to  $D$  if and only if  $\theta(X) = 0$ . Hence the distribution  $D$  is defined by  $\theta = 0$  in  $U$ . Therefore the almost contact structure  $(f, \xi, \theta)$  is associated with the hyperdistribution  $D$  with complex structure  $e^3$ .

We now take another affine normal  $C$  to  $M$  in  $U$ . Then by Lemma 1.1 we have

$$(2.6) \quad \bar{C} = -\frac{1}{\alpha}(C + A),$$

where  $\alpha$  is a non-vanishing function and  $A$  a vector field being tangent to  $M$  and belonging to  $D$ ,  $\alpha$  and  $A$  being defined in  $U$ . Thus we have

$$(2.7) \quad \bar{f} = -f + \theta \otimes \xi, \quad \bar{\xi} = -\frac{1}{\alpha}(\xi - fA), \quad \bar{\theta} = \alpha\theta,$$

where  $(\bar{f}, \bar{\xi}, \bar{\theta})$  is the almost contact structure induced in  $M$  by (2.3) and (2.5),  $C$  being replaced by  $\bar{C}$ . The change (2.7) of almost contact structures has been discussed<sup>3</sup> and is called a change of almost contact structures associated with  $D$ .

### 3. Induced affine connections :

We now assume that the ambient manifold  $\tilde{M}$  is a complex manifold of complex dimension  $n + 1$  ( $\geq 2$ ) with almost hyperbolic Hermitian structure  $\tilde{F}$ . It is well known that there is a symmetric affine connection  $\tilde{\nabla}$  satisfying  $\tilde{\nabla} \tilde{F} = 0$ , i. e.

$$(3.1) \quad \tilde{\nabla}_j \tilde{F}_i^h = 0$$

In the sequel we fix this affine connection  $\tilde{\nabla}$ .

Consider a real hypersurface  $M$  immersed in  $\tilde{M}$  and a coordinate neighborhood  $U$  of  $M$  such that  $U$  is a connected component of  $\bar{U} \cap M$ ,  $\bar{U}$  being a coordinate Neighborhood of  $\tilde{M}$ . Let  $(x^h)$  and  $(y^a)$  be coordinates in  $\bar{U}$  and in  $U$  respectively.

We assume that  $M$  is represented in  $\bar{U}$  by

$$(3.2) \quad x^h = x^h(y^a).$$

Take an affine normal  $C$  to  $M$  in  $U$  and put

$$(3.3) \quad B_b^h = \partial x^h / \partial y^a$$

in  $U$ . Then  $B_b = B_b^h \partial / \partial x^h$  and  $C = C^h \partial / \partial x^h$

form an affine  $(2n+2)$ -frame along  $U$ .

$$\text{Thus on putting } \begin{pmatrix} B_i^a \\ C_i \end{pmatrix} = - (B_b^h, C^h)^{-1}$$

we have

$$(3.4) \quad B_b^i B_i^a = \delta_b^a, \quad B_i^a C^i = 0, \quad C_i C^i = 1;$$

$$(3.5) \quad B_e^h B_i^e + C^h C_i = -\delta_i^h.$$

Thus  $B^a = B_i^a \partial x^i$  and  $C = C_i \partial x^i$  form a coframe dual to  $\{B_b, C\}$  along  $U$ .

The affine connection  $\nabla$  induced in  $U$  from  $\tilde{T}_{ji}^h$  with respect to the affine normal  $C$  has, by definition, components given by

$$(3.6) \quad T_{cb}^a = (\partial_c B_b^h + \tilde{T}_{ji}^h B_c^j B_b^i) B_h^a$$

where  $\partial_b = \partial / \partial y^b$  and  $\tilde{T}_{ji}^h$  denote components

of  $\tilde{\nabla}$  in  $\bar{U}$ . Since  $\tilde{\nabla}$  is symmetric, i.e. . . . since

$$\tilde{T}_{ji}^h = \tilde{T}_{ij}^h \nabla \text{ is also symmetric, i.e. . . . } T_{cb}^a = T_{bc}^a.$$

Thus if we define the so-called vander Waerden- Bortolotti covariant derivative of  $B_b^h$  along  $M$  by

$$(3.7) \quad \nabla_c B_b^h = -\partial_c B_b^h - \tilde{T}_{ji}^h B_c^j B_b^i + T_{cb}^a B_a^h$$

in  $U$ , then we have  $(\nabla_c B_b^h) B_b^a = 0$ , which

shows that  $\nabla_c B_b^h$  is of the form

$$(3.8) \quad \nabla_c B_b^h = h_{cb} C^h,$$

where  $h_{cb}$  are defined by

$$(3.9) \quad h_{cb} = h_{bc} = \partial_c B_b^h + (\tilde{T}_{ji}^h B_c^j B_b^i) C_h$$

and are called components of the covariant second fundamental tensor  $h$  of  $M$  with respect to the affine normal  $C$ ,  $h$  being of type  $(0, 2)$ .

Differentiating  $B_b^h B_h^a = -\delta_b^a$  covariantly along  $M$  and using (3.8) and  $C_h B_h^a = 0$ , we find  $B_{bcb}^h (\nabla_c B_h^a) = 0$ , from which

$$(3.10) \quad \nabla_c B_i^a = -H_c^a C_i$$

where  $\nabla_c B_i^a$  are defined by

$$(3.11) \quad \nabla_c B_i^a = -\partial_c B_i^a - \tilde{T}_{ji}^h B_c^j B_h^a - T_{cb}^a B_i^b$$

in  $U$  and  $H_c^a$  by

$$(3.12) \quad H_c^a = (\partial_c B_b^h + \tilde{T}_{ji}^h B_c^j C^i) B_h^a$$

The  $H_c^a$  are called components of the mixed second fundamental tensor  $H$  of  $M$  with respect to the affine normal  $C$  in  $U$ ,  $H$  being of type  $(1, 1)$ .

We next differentiate  $B_i^a C^i = 0$  covariantly along  $M$  and use (3.10). Then we obtain  $H_c^a - B_i^a (\nabla_c C^i) = 0$  from which

$$(3.13) \quad \nabla_c C^h = H_c^a B_a^h - l_c C^h,$$

where  $l_c$  are defined by

$$(3.14) \quad l_c = (\partial_c C^h + \tilde{T}_{ji}^h B_c^j C^i) C_h$$

and  $\nabla_c C^h$  by

$$(3.15) \quad \nabla_c C^h = \partial_c C^h + \tilde{T}_{ji}^h B_c^j C^i$$

in  $U$ . The  $l_c$  are called components of the third fundamental tensor  $l$  of  $M$  with respect to the

affine normal  $C$  in  $U$ ,  $l$  being of type  $(0, 1)$ . The  $l$  gives a linear connection in the one-dimensional

vector bundle  $\cup_{p \in U} \{aC_p / a \in R\}$  over  $U$ .

Finally, differentiating  $B_b^i C_i = 0$  covariantly along  $M$  and using (3.8),  $C^i B_i^a = 0$  and  $C^i C_i = 1$ , we find  $l_c - C^i (\partial_c C_i) = 0$ , from which

$$(3.16) \quad \nabla_c C_i = h_{ce} + l_c C_i \text{ where } \nabla_c C_i \text{ are defined in } U \text{ by}$$

$$(3.17) \quad \nabla_c C_i = \partial_c C_i + \tilde{T}_{ji}^h B_c^j C_h.$$

Equations (3.8) and (3.10) are those of Gauss for the real hypersurface  $M$  and equations (3.11) and (3.13) are those of Weingarten for  $M$ .

Consider a vector field  $X = X^h \partial / \partial x^h$  tangent to  $M$ . Then we have  $X^h = X^a B_a^h$ . Thus using (3.8), we have

$$(3.18) \quad \nabla_c X^h = -(\partial_c X^a) B_a^h - h_{cb} X^b C^h,$$

where we have put in  $U$ ,  $\nabla_c X^h = \partial_c X^h + \tilde{T}_{ji}^h B_c^j X^i$ ,  $\nabla_c X^a = \partial_c X^a + T_{cb}^a X^b$

Let  $(f, \xi, \theta)$  be the e almost contact structure induced in  $M$  by the affine normal  $C$  to  $M$  in  $U$ . The n (2.2) and (2.3) can be written as

$$(3.19) \quad \tilde{F}_i^h C^i = \xi^b B_b^h,$$

$$(3.20) \quad \tilde{F}_i^h B_b^i = f_b^a B_a^h - l_b C^h$$

respectively. Applying  $\nabla_c$  to (3.20) and using

$$\nabla_c \tilde{F}_i^h = B_c^j \tilde{\nabla}_j \tilde{F}_i^h = 0, \text{ we obtain}$$

$$h_{cb} (f^a B_a^h) = (\nabla_c f_b^a) B_a^h + h_{ce} f_b^e C^h - (\nabla_c \theta_b) C^h + (H_c^a B_a^h - l_c C^h),$$

where we have used (2.2), (2.3) with  $X = B_b$ , (3.16) and (3.17). Thus we obtain

$$(3.21) \quad \nabla_c f_b^a h_{cb} \xi^a - H_c^a \theta_b,$$

$$(3.22) \quad \nabla_c \theta_b = h_{ce} f_b^e + l_c \theta_b.$$

Next, applying  $\nabla_c$  to (3.19), we have in a similar way as above from which

$$(3.23) \quad \nabla_c \xi^a = f_e^a H_c^e - l_c \xi^a,$$

$$(3.24) \quad H_c^e \theta_e = -h_{ce} \xi^e.$$

Substituting (3.21), (3.22) and (3.23) into (1.3) and using (3.24), we obtain

$$(3.25) \quad S_{cb}^a = (-H_e^a f_c^e + f_e^a H_c^e - l_c \xi^a) \theta_b + (-H_e^a f_b^e + f_e^a H_b^e - l_b \xi^a) \theta_c,$$

$$(3.26) \quad G_{cb} = h_{cb} + f_c^e f_b^d h_{ed} - h_{be} \xi^e \theta_c + f_c^e l_e \theta_b,$$

$$(3.28) \quad Q_b = -l_b + (l_e \xi^e) \theta_b + h_{ed} \xi^e f_b^d.$$

When a hyperdistribution D with almost hyperbolic Hermitian structure J is given on a manifold of odd dimension and when  $S_{cb}^a \equiv 0, (\text{mod } \theta_c, \theta_b)$  is satisfied for an almost para contact structure  $(f, \xi, \theta)$  associated with D, the D is said to be torsionless. Thus we have

from (3.25).

*Proposition 3.1:* For any real hypersurface M of an almost hyperbolic Hermitian manifold the induced hyperdistribution D of M with almost hyperbolic Hermitian structure J is always torsionless<sup>12-14</sup>.

Equations (3.26) imply

*Proposition 3.2:* For any real hypersurface M of a almost hyperbolic Hermitian manifold, the Levi-tensor G of an almost para contact structure  $(f, \xi, \theta)$  induced in M has components of the form<sup>6-11</sup>

$$(3.29) \quad G_{cb} = h_{cb} + f_c^e f_b^d h_{ed} \pmod{\theta_c, \theta_b}$$

in U, when an affine normal C to M is given in a coordinate neighborhood U of M. Proposition 3.2 implies that  $g(X, Y) = g(Y, X)$ ,  $g(JX, JY) + g(X, Y) = 0$  for any vector fields X and Y belonging to the hyperdistribution D with hyperbolic RAC Structure J. Equations (3.25) imply

*Proposition 3.3:* Let  $(f, \xi, \theta)$  be an almost para contact structure induced on a real hypersurface M of an almost hyperbolic Hermitian manifold by giving an affine normal C to M in a coordinate neighborhood U of M. Then  $(f, \xi, \theta)$  is normal if and only if

$$(3.30) \quad -H_e^a f_b^e + f_e^a H_b^e - l_b \xi^a \equiv 0, (\text{mod } \theta_b).$$

We take another affine normal  $\bar{C}$  to M in U and assume  $\bar{C}$  is given by (2.6).

Denote by  $\bar{\nabla}, \bar{l}, \bar{h}$  and  $\bar{H}$  respectively the induced affine connection, the third fundamental

tensor, the covariant and the mixed second fundamental tensors of M in U, which are determined by (3.6), (3.14), (3.9) and (3.12) in terms of  $\bar{C}$ .

Then components  $\bar{T}_{ca}^b$  of  $\bar{\nabla}$ ,  $\bar{h}_{cb}$  of  $\bar{h}$ ,  $\bar{H}_b^a$  of  $\bar{H}$  and  $\bar{l}_b$  of  $\bar{l}$  are respectively given by

$$\begin{aligned} \bar{T}_{cb}^a &= T_{cb}^a + h_{cb} A^a, & \bar{h}_{cb} &= \alpha h_{cb}, \\ (3.31) \quad \bar{H}_b^a &= -\frac{1}{\alpha} [H_b^a + \nabla_b A^a - (l_b + h_{be} A^e) A^a], \\ \bar{l}_b &= -(l_b + h_{be} A^e) + \nabla_b \log|\alpha|, \end{aligned}$$

where  $\alpha$  is a non-vanishing function and  $A = A^a B_a^h \partial / \partial x^h$  is a vector field belonging to D, both being defined in U. To obtain (3.31), we have used (2.6),  $\bar{B}_b^h = B_b^h$  and

$$(3.32) \quad \bar{B}_i^a = B_i^a + A^a C_i, \quad \bar{c}_i = C_i,$$

where  $\begin{pmatrix} \bar{B}_i^a \\ \bar{C}_i \end{pmatrix} = -(\bar{B}_a^h, \bar{C}^h)^{-1}$ .

Proposition 3.2 and  $\bar{h}_{cb} = \alpha h_{cb}$  appearing in (3.21) imply the following well known Proposition<sup>1-5</sup>:

*Proposition 3.4.* Let  $(f, \xi, \theta)$  and  $(\bar{f}, \bar{\xi}, \bar{\theta})$  be two almost para contact structures induced on a real hypersurface M and assume that they are related to each other by (2.7). Then  $\bar{G}_{cb} = \alpha G_{cb} \pmod{\theta_c, \theta_b}$ ,  $\alpha$  being a non-vanishing function, where  $\bar{G}_{cb}$  and  $G_{cb}$

are respectively components of the Levi tensors of  $(f, \xi, \theta)$  and  $(\bar{f}, \bar{\xi}, \bar{\theta})$ .

Proposition 3.4 shows that the restriction  $G_D$  of the Levi tensor G to D is determined up to a non-vanishing factor. Thus  $G_D$  is sometimes called the Levi tensor of the induced hyperdistribution D with almost hyperbolic Hermitian structure. When G is of the maximum rank 2n everywhere in M, the real hypersurface M is said to be non-degenerate. By P3.1, for any real hypersurface M of an almost hyperbolic Hermitian manifold the hyperdistribution D of M with almost hyperbolic Hermitian structure is torsionless. This fact means that any real hypersurface M admits a pseudo-conformal structure when M is non-degenerate<sup>1,2,5</sup>.

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