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New sort of operators in Nano Ideal Topology

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Abstract

This paper is devoted to nano ideal space and stipulated the concept of nano local function. Based on the Kuratowski's closure operator the several forms of nano approximations were inquired in the nano ideal space. Different types of weak forms of nano open sets like nano semi *I*-open sets and nano α*I*-open sets were explored in aspects of distinct approximations. Further more, to emphasize our nano ideal space some examples are considered, which are efficacious for implementing this hypothesis in practical applications. Conclusively, We portayed the relation among the weaker form of nano open sets in nano ideal space.

Key words: Nano topology, Nano local function, Nano closure operator, Nano semi *I* -open sets, nano α*I*-open sets.

1 Introduction

The concept of ideal in topological space was first introduced by Kuratowski[5]. They also have defined local function in ideal topological space. Further Hamlett and Jankovic[3] investigated further properties of topological space. Lellis Thivagar *et al*.[6] established nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano-open sets. The topology recommended here is named so because of its size, Since it has atmost five elements in it. In this paper, we introduce ideal in nano topological space, and define a nano local function for each subset with respect to *I* and $\tau_R(X)$. In nano ideal space through nano closure operator we have endeavoured the several weaker forms of nano open sets. Eventually, we depict the link among the weaker form of nano open sets in the ideal space.

2 Preliminaries :

The following recalls requisite ideas

and preliminaries necessitated in the sequel of our work.

Definition 2.1[6]*:* Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

(i) The Lower appproximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R X$. That is, $L_R(X) = \left\{\bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}\right\}$, where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \left\{\bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}\right\}$

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified as neither as X nor as not –X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2[6] **Error! Reference source not found.***:* Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. τ RX satisfies the following axioms:

(i) U and $\emptyset \in \tau_R(X)$

(ii) The union of elements of any subcollection of $\tau_R X$ is in $\tau_R X$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R X$ is in $\tau_R X$.

That is, $\tau_R X$ forms a topology on U called as the nano topology on U with respect to X. We call $\{U, \tau_R(X)\}$ as the nano topological space.

Definition 2.3 [3] *: An ideal I on a topological space is a non-empty collection of subsets of X which satisfies*

(i) $A \in B$ and $B \subset A \Rightarrow B \in I$.

(ii) $A \in I$ and $B \in I \Rightarrow A \cup B \in I$.

3 $(A)^{*N}$ operator

In this section we introduced a operator, called the nano local function and the nano closure operator in nano ideal space.

Definition 3.1: A nano topological space $\{U, \tau_R(X)\}$ with an ideal I on U is called an nano ideal topological space or nano ideal space and denoted as $\{U, \tau_R(X), I\}$.

Definition 3.2 : Let $\{U, \tau_R(X), I\}$ be a nano ideal topological space. A set operator $(A)^{*N} : P(U) \to P(U)$, is called the nano local function of I on U with respect to I on $\tau_R(X)$ is defined as $(A)^{*N} = \{x \in : U \cap A \notin I;$ for every $U \in \tau_R(X)\}$ and is denoted by $(A)^{*N}$, where nano closure operator is defined as $Ncl^*A) = A \cup (A)^{*N}$

Theorem 3.3 : Let $\{U, \tau_R(X), I\}$ be *an nano ideal topological space and let A and B be subsets of U, then*

(i) $\phi^{*N} = \phi$

(ii) $A \subset B \to A^{*N} \subset B^{*N}$

(iii) For another $J \supseteq I$ *on U,* $A^{*N}(J) \subset A^{*N}(I)$

(iv) $A^{*N} \subset cl^{*}(A)$

(v) A^{*N} *is a nano closed set*

(vi) $(A^{*N})^{*N} \subset A^{*N}$

(vii) $A^{*N} \cup B^{*N} = (A \cup B)^{*N}$

(viii) $(A \cap B)^{*N} = A^{*N} \cap B^{*N}$

(ix) For every nano open set V,
$V \cap (V \cap A)^{*N} \subset (V \cap A)^{*N}$

(x) For $I \in I$, $(A \cup I)^{*N} = A^{*N} = (A - I)^{*N}$

Proof: i) It is obvious from the defn. of nano local function.

(ii) Let $x \in A^{*N}$, then for every $U \in \tau_R(X)$, $U \cap A \notin I$, since $U \cap A \subset U \cap B$, then $U \cap B \notin I$. This implies that $x \in B^{*N}$.

(iii) Let $x \in A^{*N}(J)$, then for every $U \in \tau_R(X)$, $U \cap A \notin J$, this implies that $U \cap A \notin I$, so $x \in A^{*N}(I)$. Hence $A^{*N}(J) \subset A^{*N}(I)$.

(iv) Let $x \in A^{*N}$, then for every $U \in \tau_R(X)$, $U \cap A \notin I$, since $U \cap A \neq \phi$. Hence $x \in Ncl(A)$.

(v) From the defn of nano neighbourhood, each nano neighbourhood M of U contains a $U \in \tau_R(X)$. Now $A \cap M \in I$, then for $A \cap U \subset A \cap M$, $A \cap U \in I$. It follows that $U - A^{*N}$ is the union of nano open sets. Since arbitrary union of nano open sets is a nano open set, so $U - A^{*N}$ is a nano open set. Thus A^{*N} is a nano closed set.

(vi) $(A^{*N})^{*N} \subset Ncl(A^{*N}) = A^{*N}$, since A^{*N} is a nano closed set.

(vii) we know that $A \subset A \cup B$ and $B \subset A \cup B$, then from (ii) $(A^{*N}) \subset (A \cup B)^{*N}$ and $(B^{*N}) \subset (A \cup B)^{*N}$. Hence, $A^{*N} \cup B^{*N} \subset (A \cup B)^{*N}$.

(ix) Since $V \cap A \subset A$, then $(V \cap A)^{*N} \subset A^{*N}$. so $V \cap (V \cap A)^{*N} \subset V \cap A^{*N}$.

(x) Since $A \subset A \cup I$, then $A^{*N} \subset (A \cup I)^{*N}$. Let $x \in (A \cup I)^{*N} \notin I$. Then for every $U \in \tau_R(X)$, $U \cap (A \cup I) \notin I$. This implies that $U \cap A \notin I$. If possible suppose that $U \cap A \in I$, again $U \cap I \subset I$ implies $U \cap I \in I$, $U \cap (A \cap I) \cap I$, a contradiction. Hence $x \in A^{*N}$ and $(A \cup I)^{*N} \subset A^{*N}$.
Thus $(A \cup I)^{*N} = A^{*N}$.

Theorem 3.4 : Let $\{U, \tau_R(X), I\}$ *be an nano ideal topological space and A be a subset of U, If* $A \subset A^{*N}$*, then* $A^{*N} = Ncl(A^{*N}) = Ncl(A) = Ncl^{*}(A)$.

Proof: Let A be any subset of U, we have $A^{*N} = cl(A^{*N}) \subset Ncl(A)$, $A \subset A^{*N}$ implies that $cl(A) \subset cl(A^{*N})$ and so $A^{*N} = cl(A^{*N}) = Ncl(A)$. Clearly for every subset A of U, $Ncl^*(A) \subset Ncl(A)$. Let $x \notin Ncl^*(A)$, then there exists a nano open set G containing "x" such that $G \cap A = \phi$, There exists $V \in \tau_R(X)$ and $\in I$ such that $x \in V - I \subset G$. $G \cap A = \phi \Rightarrow (V - I) \cap A = \phi \Rightarrow (V \cap A) - I = \phi \Rightarrow (V \cap A)^{*N} - I^{*N} = \phi \Rightarrow (V \cap A)^{*N} - I^{*N} = \phi \Rightarrow (V \cap A)^{*N} = \phi \Rightarrow V \cap A^{*N} = \phi \Rightarrow (V \cap A)$

$= \phi$. Since V is an nano open set containing x, $x \notin Ncl(A)$ and so $Ncl(A) \subset Ncl^*(A)$. Hence $Ncl(A) = Ncl^*(A)$.

4 Nano Closure operator relying on approximations :

In this section we find the nano closure operator based on their approximations.

Theorem 4.1 : Let $\{U, \tau_R(X), I\}$ be a nano ideal topological space and if $L_R(X) = U_R(X) = X$, then

$$Ncl^*[L_R(X)] = \begin{cases} L_R(X) & \text{if } L_R(X) \in I \\ U & \text{if } L_R(X) \notin I \end{cases}$$

Proof: Let $L_R(X) = U_R(X) = X$, then $\tau_R(X) = \{U, \phi, L_R(X)\}$.

(i) If $L_R(X) \in I$ then by the defn. of nano local function, for each $x \in U$ a nano open set V containing x is such that $L_R(X)$ or U. Therefore $V \cap L_R(X) = L_R(X) \in I$. Hence $(L_R(X))^{*N} = \phi$ and also $Ncl^*[L_R(X)] = L_R(X)$.

(ii) If $L_R(X) \notin I$, for each $x \in U$, an nano open set V containing x is such that $L_R(X)$ or U. Since $L_R(X) \subseteq U$ and for all x, $V \cap L_R(X) = L_R(X) \notin I$. Therefore $[L_R(X)]^{*N} = U$ and $Ncl^*[L_R(X)] = U$.

Theorem 4.2 : Let $\{U, \tau_R(X), I\}$ be a nano ideal topological space and if $L_R(X) = \phi$, $U_R(X) \neq \phi$ then

$$Ncl^*[U_R(X)] = \begin{cases} U_R(X) & \text{if } U_R(X) \in I \\ U & \text{if } U_R(X) \notin I \end{cases}$$

Proof: Let $L_R(X) = \phi$, $U_R(X) \neq \phi$, then $\tau_R(X) = \{U, \phi, U_R(X)\}$.

(i) If $U_R(X) \in I$ then by the defn. of nano local function, for each $x \in U$ a nano open set V containing x is such that $U_R(X)$ or U. Therefore $V \cap U_R(X) = U_R(X) \in I$. Hence $(U_R(X))^{*N} = \phi$ and also $Ncl^*[U_R(X)] = U_R(X)$.

(ii) If $U_R(X) \notin I$, for each $x \in U$, an nano open set V containing x is such that $U_R(X)$ or U. Since $U_R(X) \subseteq U$ and for all x, $V \cap U_R(X) = U_R(X) \notin I$. Therefore $[U_R(X)]^{*N} = U$ and $Ncl^*[U_R(X)] = U$.

Theorem 4.3: Let $\{U, \tau_R(X)\}$ be a nano topological space, for every ideal I on U and if $L_R(X) \neq \phi$ $U_R(X) = U$, then $Ncl^*(L_R(X)) = L_R(X)$ and $Ncl^*[B_R(X)] = B_R(X)$

Proof: If $L_R(X) \neq \phi$ $U_R(X) = U$, then $\tau_R(X) = \{U, \phi, L_R(X), B_R(X)\}$.
For every ideal on U and for each $x \in U$ a nano open set V containing 'x' is such that either $L_R(X)$, $B_R(X)$ or ϕ. Then for each $x \in U$, $V \cap L_R(X) = L_R(X)$ or ϕ, $V \cap B_R(X) = B_R(X)$ or ϕ. Hence $Ncl^*(L_R(X)) = L_R(X)$ and $Ncl^*[B_R(X)] = B_R(X)$.

Theorem 4.4: Let $\{U, \tau_R(X)\}$ be a nano topological space, and $L_R(X) = U_R(X)$ Where $U_R(X) \neq U$, $L_R(X) \neq \phi$ with an ideal $I = \{\phi, L_R(X)\}$, then
(i) $Ncl^*[L_R(X)] = L_R(X)$
(ii) $Ncl^*[U_R(X)] = U$
(iii) $Ncl^*[B_R(X)] = [L_R(X)]^C$

Proof: Let $L_R(X) \neq U_R(X)$, where $U_R(X)$

$\neq U$ and $LR(X) \neq \phi$ Then $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and if $L_R(X) \in I$, for each $x \in U$, a nano open set V containing x is such that U or $L_R(X)$ or $U_R(X)$ or $B_R(X)$.

(i). Now $V \cap L_R(X) = L_R(X)$ *or* $\phi \in I$. Hence $[L_R(X)]^{*N} = \phi$ and also $Ncl^*[L_R(X)] = L_R(X)$

(ii). We know that $L_R(X) \subseteq [U_R(X)]$ and $B_R(X) \subseteq [U_R(X)]$, $U_R(X) \subseteq U$, we have $V \cap U_R(X) = U_R(X)$ *or* $L_R(X)$ *or* $B_R(X)$. Since $L_R(X) \in I$ for all x, $V \cap U_R(X) = U_R(X)$ $B_R(X) \notin I$. Hence $[U_R(X)]^{*N} = \{x : x \in U; x \notin L_R(X) = [L_R(X)]^c$ and also $Ncl^*[U_R(X)] = U_R(X) \cup [L_R(X)]^c = U$.

(iii). $V \cap B_R(X) = B_R(X)$ or ϕ. here $B_R(X) \notin I$, But $V = L_R(X) \in I$, which implies $V \cap B_R(X) = \phi$. Hence $[B_R(X)]^{*N} = \{x : x \in U, x \notin L_R(X)\} = [L_R(X)]^c$ and also $Ncl^*[B_R(X)] = B_R(X) \cup [L_R(X)]^c = [L_R(X)]^c$.

Theorem 4.5: Let $\{U, \tau_R(X)\}$ be a nano topological space, and $L_R(X) \neq U_R(X)$, where $U_R(X) \neq U$, $L_R(X) \neq \phi$ with an ideal $I = \{\phi, U_R(X)\}$, then

(i) $Ncl^*[L_R(X)] = L_R(X)$

(ii) $Ncl^*[U_R(X)] = U_R(X)$

(iii) $Ncl^*[B_R(X)] = B_R(X)$

Proof: Let $L_R(X) \neq U_R(X)$, where $U_R(X) \neq U$ and $L_R(X) \neq \phi$. Then $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ We know that $L_R(X) \subseteq U_R(X)$ and $B_R(X) \subseteq U_R(X)$, $U_R(X) \in U$ and by defn. of ideal we have $L_R(X)$, $B_R(X) \in I$. For each $x \in U$, a nano open set V containing x is such that U *or* $L_R(X)$ *or* $U_R(X)$ *or* $B_R(X)$.

(i). Now $V \cap L_R(X) = L_R(X)$ *or* $\phi \in I$. Hence $[L_R(X)]^{*N} = \phi$ and also $Ncl^*[L_R(X)] = L_R(X)$.

(ii). Now consider $V \cap U_R(X) = U_R(X)$ *or* $L_R(X)$ *or* $BR(X)$, which concludes that $V \cap U_R(X) \in I$. Hence $[U_R(X)]^{*N} = \phi$ and also $Ncl^*[U_R(X)] = U_R(X)$.

(iii). $V \cap B_R(X) = B_R(X)$ or $\phi \in I$. Hence $[B_R(X)]^{*N} = \phi$ and also $Ncl^*[B_R(X)] = B_R(X)$.

Theorem 4.6: Let $\{U, \tau_R(X)\}$ be a nano topological space and $L_R(X) \neq U_R(X)$, where $U_R(X) \neq U$, $L_R(X) \neq \phi$ with an ideal $I = \{\phi, B_R(X)\}$, then

(i) $Ncl^*[L_R(X)] = [B_R(X)]^C$

(ii) $Ncl^*[U_R(X)] = U$

(iii) $Ncl^*[B_R(X)] = B_R(X)$

Proof: Let $L_R(X) = U_R(X)$, where $U_R(X) \neq U$, $L_R(X) \neq \phi$. Then $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and if $B_R(X) \in I$, for each $x \in U$, a nano open set V containing x is such that U *or* $L_R(X)$ *or* $U_R(X)$ *or* $B_R(X)$.

(i). Now $V \cap L_R(X) = L_R(X)$ *or* $\phi \in I$. here $L_R(X) \notin I$, but $V = B_R(X) \in I$, which implies that $V \cap L_R(X) = \phi$. Hence $[L_R(X)]^{*N} = \{x \in U : x \notin B_R(X) = [B_R(X)]^c$ and also $Ncl^*[L_R(X)] = L_R(X) \cup [B_R(X)]^c = [B_R(X)]^c$.

(ii). Now $V \cap U_R(X) = U_R(X)$ *or* $L_R(X)$ *or* $B_R(X)$ since $B_R(X) \in I$, $[U_R(X)]^{*N} = \{x \in U : x \notin B_R(X)\} = [B_R(X)]^c$ and also $Ncl^*[U_R(X)] = U_R(X) \cup [B_R(X)]^c = U$.

(iii). $V \cap B_R(X) = B_R(X)$ *or* $\phi \in I$. Hence

$[B_R(X)]^{*N} = \phi$ and also
$Ncl^*[B_R(X)] = B_R(X)$.

Theorem 4.7 : For every ideal I such that $L_R(X)$, $U_R(X)$; $B_R(X) \notin I$ in a nano topological space $\{U \tau_R(X)\}$ and $L_R(X) \neq U_R(X)$, where $U_R(X) \neq U$, $L_R(X)) \neq \phi$ then
(i) $Ncl^*[L_R(X)] = [U_R(X)]C$
(ii) $Ncl^*[U_R(X)] = U$
(iii) $Ncl^*[B_R(X)] = [L_R(X)]C$

Proof: Let $L_R(X)) \neq U_R(X)$, where $U_R(X)) \neq U$, $L_R(X) \neq \phi$. Then $\tau_R(X) = \{U, \phi, L_R(X); U_R(X), B_R(X)\}$, for each $x \in U$, a nano open set V containing x is such that U *or* $L_R(X)$ *or* $U_R(X)$ *or* $B_R(X)$.
(i). Now $V \cap L_R(X) = L_R(X)$ *or* $\phi \in I$. here $L_R(X) \notin I$, but $V = B_R(X)$, which implies that $(V \cap L_R(X)) = \phi$. Hence $[L_R(X))]^{*N} = \{x \in U: x \notin B_R(X)\} = [B_R(X)]c$ and also $Ncl^*[L_R(X)] = L_R(X) \cup [B_R(X)]^c = [B_R(X)]^c$.
(ii). Now $V \cap U_R(X) = U_R(X)$ *or* $L_R(X)$ *or* $B_R(X)$. For each $x \in U$, $V \cap U_R(X) \notin I$.
Hence $[U_R(X)]^{*N} = U$ and also $Ncl^*[U_R(X)] = U$.
(iii). $V \cap B_R(X) = B_R(X)$ or $\phi \in I$. here $B_R(X) \notin I$, But $V = L_R(X)$, which implies $V \cap B_R(X) = \phi$. Hence $[B_R(X)]^{*N} = [L_R(X)]^c$ and also
$Ncl^*[B_R(X)] = B_R(X) \cup [L_R(X)]^c = [L_R(X)]c$.

5. Nano SemiI-open sets :

In this section we have revealed that the choice of an ideal has an impact on Nano Semi*I*-open sets.

Definition 5.1 : Let $\{U, \tau_R(X), I\}$ be a nano ideal topological space and $A \subseteq U$, then
A is said to be Nano Semi ideal open or Nano SemiI-open if $A \subseteq Ncl^*[NInt(A)]$.

Theorem 5.2 : If in a nano ideal topological space $\{U, \tau_R(X), I\}$, $L_R(X) = U_R(X)$
(i) If $L_R(X) \notin I$, then ϕ and those sets for which $A \supseteq L_R(X)$ are the only nano semi I-open sets in U.
(ii) If $L_R(X) \in I$, then I, ϕ, $L_R(X)$ are the only nano semi I-open sets in U.

Proof: $\tau_R(X) = \{U, \phi, sL_R(X)\}$ and $L_R(X) \notin I$. If A is a non-empty subset of U such that $A \subset L_R(X)$, then $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi*I*-open. If $A \supseteq L_R(X) \notin I$, then $Ncl^*[NInt(A)] = U$. Therefore $A \subseteq Ncl^*[NInt(A)]$. Hence A is nano semi*I*-open. Thus ϕ and sets containing $L_R(X)$ are the only nano semi*I*-open sets in U.
(ii) If $L_R(X) \in I$ and let A be a non-empty subset of U such that $A \subset L_R(X)$, then $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi $I - open$. If $A \supset L_R(X)$, then $Ncl^*[NInt(A)] = Ncl^*[L_R(X)] = L_R(X)$. Hence A is not nano semi *I*-open. If $A = L_R(X) \in I$, then $Ncl^*[NInt(A)] = Ncl^*[L_R(X)] = L_R(X) = A$. Hence A is not nano semi I - *open*. Thus the U, ϕ, $L_R(X)$ are the only nano semi *I*-open sets in U.

Theorem 5.3 : If in a nano ideal topological space $\{U, \tau_R(X), I\}$, $L_R(X) = \phi$ and $U_R(X) \neq U$
(i) If $U_R(X) \notin I$, then ϕ and those sets for which

$A \supseteq U_R(X)$ are the only nanosemi I-open sets in U.
(ii) If $U_R(X) \in I$, then I, ϕ, $U_R(X)$ are the only nano semiI-open sets in U.

Proof: $\tau_R(X) = \{U, \phi, U_R(X)\}$ and $U_R(X) \notin I$. If A is a non-empty subset of U such that $A \subset U_R(X)$, then $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi$I$-open. If $A \supseteq U_R(X) \notin I$, then $Ncl^*[NInt(A)] = U$. Therefore $A \subseteq Ncl^*[NInt(A)]$. Hence A is nano semi I-open. Thus ϕ and sets containing $U_R(X)$ are the only nano semi$I$-open sets in U.
(ii) If $U_R(X) \in I$ and let A be a non-empty subset of U such that $A \subset U_R(X)$, then $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi$I$ - *open*. If $A \supset U_R(X)$, then $Ncl^*[NInt(A)] = Ncl^*[U_R(X)] = U_R(X)$. Hence A is not nano semi$I$-open. If $A = U_R(X) \in I$, then $Ncl^*[NInt(A)] = Ncl^*[U_R(X)] = U_R(X) = A$. Hence A is nano semi$I$-*open*. Thus the U, ϕ, $U_R(X)$ are the only nano semi$I$-open sets in U.

Theorem 5.4 : Let $\{U, \tau_R(X)\}$ be a nano topological space,for every ideal I on U and if $L_R(X) \neq \phi$, $U_R(X) = U$, then U, ϕ, $L_R(X)$ $B_R(X)$ are the only nano semi I-open sets in U.

Proof: $\tau_R(X) = \{U, \phi, L_R(X), B_R(X)\}$. Let A be a non-empty subset of U, If $A \subset L_R(X)$, $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi I-*open*. If $A \supseteq L_R(X) \notin I$, then $Ncl^*[NInt(A)] = L_R(X)$. Therefore A $\not\subset Ncl^*[NInt(A)]$. Hence A is not nano semi I - *open* in U. If $A = L_R(X)$, then $Ncl^*[NInt(A)] = L_R(X)$. Therefore $A \subseteq Ncl^*[NInt(A)]$. Hence A is nano semi I - *open* in U.
If $A \subset B_R(X)$, $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi I - *open*. If $A \supseteq B_R(X) \notin I$, then $Ncl^*[NInt(A)] = B_R(X)$. Therefore $A \subseteq Ncl^*[NInt(A)]$. Hence A is not nano semi I - *open* in U. If $A = B_R(X)$, then $Ncl^*[NInt(A)] = B_R(X)$. Therefore $A \subseteq Ncl^*[NInt(A)]$. Hence A is nano semi I - *open* in U.

If A has atleast one element of $L_R(X)$ and atleast one element of $B_R(X)$, then $Ncl^*[NInt(A)] = \phi$ or $L_R(X)$ or $B_R(X)$ and Hence $A \not\subset Ncl^*[NInt(A)]$. Therefore A is not nano semi$I$-*open* in U. Thus, U, ϕ, $L_R(X)$, $B_R(X)$ are the only nano semi$I$-open sets in U.

Theorem 5.5: If $\tau_R(X) = \{U, \phi, L_R(X)$ $U_R(X)$, $B_R(X)\}$ be a nano topological space with an ideal $I = \{\phi, L_R(X)\}$ then the nano semi I-open sets are U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$, any set $\supset U_R(X)$ and $B_R(X) \cup B$, where $B \subseteq [U_R(X)]^c$.

Proof: $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and let A be a nonempty proper subset of U.
If $A \subset L_R(X)$, then $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi I-open in U. If $A = L_R(X)$, then $Ncl^*[NInt(A)] = L_R(X)$ and Hence $A \subseteq Ncl^*[NInt(A)]$. Therefore $L_R(X)$ is nano semi I-open in U.
If $A \subset B_R(X)$, then $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi I-open in U. If $A = B_R(X)$, then $Ncl^*[NInt(A)] = [L_R(X)]^c = B_R(X) \cup [U_R(X)]^c$

and Hence $A \subseteq Ncl^*[NInt(A)]$. Therefore $B_R(X)$ is nano semi I-open in U. Since $L_R(X)$ and $B_R(X)$ are nano semi I-open $L_R(X) \cup B_R(X) = U_R(X)$ is also nano semi I-open in U.

Let $A \subset U_R(X)$ such that A has atleast one element each of $L_R(X)$ and $B_R(X)$. Then $NInt(A) = \phi$ or $[L_R(X)]^c$ and Hence $A \not\subset Ncl^*[NInt(A)]$. Therefore A is not nano Semi I-open in U. If $A \supset U_R(X)$, then $Ncl^*[NInt(A)] = Ncl^*[U_R(X)] = U$ and Hence $A \subseteq Ncl^*[NInt(A)]$. Therefore A is nano semi I-open.

If A has a single element of each of $L_R(X)$ and $B_R(X)$ and atleast one element of $[U_R(X)]^c$, then $Ncl^*[NInt(A)] = \phi$ and Hence A is not nano semi I-open in U. Similarly, when A has a single element of $L_R(X)$ and atleast one element of $[U_R(X)]^c$ or a single element of $B_R(X)$ and atleast one element of $[U_R(X)]^C$ then $Ncl^*[NInt(A)] = \phi$ and Hence A is not nano semi I-open in U. When $A = L_R(X) \cup B$, where $B \subseteq [U_R(X)]^c$, then $Ncl^*[NInt(A)] = \phi$ and Hence A is not nano semi I-open in U. when $A = B_R(X) \cup B$, where $B \subseteq [U_R(X)]^c$ then $Ncl^*[NInt(A)] = Ncl^*[B_R(X)] = B_R(X) \cup [U_R(X)]^c \supseteq A$ and Hence A is nano semi I-open in U.

Thus the nano semi I-open sets are U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$, any set $\supset U_R(X)$ and $B_R(X) \cup B$, where $B \subseteq [U_R(X)]^c$.

Theorem 5.6: Let $\{U, \tau_R(X)\}$ be a nano topological space, and $L_R(X) \neq U_R(X)$, where $U_R(X) \neq U$, $L_R(X) \neq \phi$ with an ideal $I = \{\phi, U_R(X)\}$ then U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$ are the only nano semi I-open sets in U.

Proof: $\tau_R(X) = \{U, \phi, L_R(X)), U_R(X), B_R(X)\}$. Let A be a non-empty proper subset of U.

If $A \subset L_R(X)$, then $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi I-open in U. If $A = L_R(X)$, then $Ncl^*[NInt(A)] = L_R(X)$, and Hence $A \subseteq Ncl^*[NInt(A)]$. Therefore $L_R(X)$ is nano semi I-open in U. If $A \supset L_R(X)$, then $Ncl^*[NInt(A)] = L_R(X) \subset A$. Therefore A is not nano semi I-open in U.

If $A \subset U_R(X)$, then $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi I-open in U. If $A = U_R(X)$, then $Ncl^*[NInt(A)] = U_R(X)$ and Hence $A \subseteq Ncl^*[NInt(A)]$. Therefore $U_R(X)$ is nano semi I-open in U. If $A \supset U_R(X)$, then $Ncl^*NInt(A)] = U_R(X) \subset A$. Therefore A is not nano semi I-open in U.

If $A \subset B_R(X)$, then $Ncl^*[NInt(A)] = \phi$. Therefore A is not nano semi I-open in U. If $A = B_R(X)$, then $Ncl^*[NInt(A)] = B_R(X)$ and Hence $A \subseteq Ncl^*[NInt(A)]$. Therefore $B_R(X)$ is nano semi I-open in U. If $A \supset B_R(X)$, then $Ncl^*[NInt(A)] = B_R(X) \subset A$. Therefore A is not nano semi I-open in U.

If A has a single element of each of $L_R(X)$ and $B_R(X)$ and atleast one element of $[UR(X)]^c$, then $Ncl^*[NInt(A)] = \phi$ and Hence A is not nano semi$I$-open in U. similarly, when A has a single element of $L_R(X)$ and atleast one element of $[UR(X)]^c$ or a single element of $B_R(X)$ and atleast one element of $[U_R(X)]^c$, then $Ncl^*[NInt(A)] = \phi$ and Hence A is not nano semi I-open in U.

Thus U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$ are the only nano semi I-open sets in U.

Proposition 5.7 : If $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ be a nano topological space with an ideal $I = \{\phi, B_R(X)\}$ then the nano semi I-open sets are U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$, any set $\supset UR(X)$ and $L_R(X) \cup B$, where $B \subseteq [UR(X)]^c$.

Proposition 5.8 : For every ideal I such that $L_R(X)$, $U_R(X)$, $B_R(X) \notin I$ in a nano topological space $\{U, \tau_R(X)\}$ and $L_R(X) \neq U_R(X)$, where $U_R(X) \neq U$, $L_R(X)) \neq \phi$ then the nano semi I-open sets are U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$, any set $\supset U_R(X)$ and $L_R(X) \cup B$, $B_R(X) \cup B$ where $B \subseteq [U_R(X)]^c$.

6 Characterization of nano semiI-open sets:

In this section we have interpreted the properties of nano semi I-open sets based on its approximations.

Theorem 6.1 : In a nano ideal space, If A and B are nano semi I-open in U, then $A \cup B$ is also nano semi I -open in U.

Proof: If A and B are nano semi I-open in U, then $A \subseteq Ncl^*[NInt(A)]$ and $B \subseteq Ncl^*[NInt(B)]$. Consider $A \cup B = Ncl^*[NInt(A)] \cup Ncl^*[NInt(B)] = Ncl^*[NInt(A) \cup NInt(B)]$. Therefore $A \cup B \subseteq Ncl^*[NInt(A \cup B)]$. Hence $A \cup B$ is nano semi$I$-open.

Remark 6.2 : If A and B are nano semi I-open in U, then $A \cap B$ is not nano semi I-open, which can be shown by the following example.

Example 6.3 : Let U = {a, b, c, d, e} and U /R = {{a}, {b, c}, {d, e}} and X = {b, c, d} $\subseteq$ U. Then in the nano ideal space $\tau_R(X)$ = {U, ϕ, {b,c}, {b, c, d,e},{d, e}} with an nano ideal I = {ϕ}, the nano semi I-open sets in U are U, ϕ, {b,c}, {b,c d, e},{d,e}, {a, b, c}, {a, d, e}. The sets A= {a,b, c} and B={a, d, e} are nano semi I-open set, but $A \cap B$ = {a} is not nano semi I-open in U.

Theorem 6.4: If in a nano ideal space $\{U, \tau_R(X), I\}$, $L_R(X) = U_R(X)$ and $L_R(X) \notin I$ then $A \cap B$ is nano semi I-open in U, whenever A and B are nano semi I-open in U.

Proof: If $L_R(X) = U_R(X)$ and $L_R(X) \notin I$ then ϕ and any set containing $L_R(X)$ is nano semi I-open. Let A and B$\neq\phi$ nano semi I-open sets in U, then $A \supseteq L_R(X)$ and $B \supseteq L_R(X)$ and $A \cap B \supseteq L_R(X)$ and Hence $A \cap B$ is nano semi I-open.

Theorem 6.5 : If in a nano ideal space $\{U, \tau_R(X), I\}$, $L_R(X) = \phi$, $U_R(X) \neq U$ and $U_R(X) \notin I$ then $A \cap B$ is nano semiI-open in U, whenever A and B are nano semiI-open in U.

Proof: If A and B $\neq\phi$ nano semi$I$-open sets in U, then $A \supseteq U_R(X)$ and $B \supseteq U_R(X)$ and $A \cap B \supseteq U_R(X)$ and Hence $A \cap B$ is nano semi$I$-open.

Theorem 6.6 : If A and B are nano semiI-closed, then $A \cap B$ is nano semi I-closed in U.

Proof: Since A and B are nano semi *I*-closed in *U*, A^c and B^c are nano semi *I*-open and Hence $A^c \cup B^c = (A \cap B)^c$ is nano semi *I*-open in *U*. Therefore, $A \cap B$ is nano semi *I*-closed in *U*.

Example 6.7 : Let U = {a, b, c, d} and U / R = {{a},{b}, {c,d}} and X = {b, d} $\subseteq$ U. Then in the nano ideal space $\tau_R(X)$= {U, ϕ, {b}, {b, c, d}, {c, d}} with an nano ideal I = {ϕ}, {a}} the nano semi I-closed sets in U are U, ϕ, {a,c,d}, {a}, {a, b}, {c,d}, {b} {a, d,e}. The sets A= {b} and B={c, d} are nano semi I-open set, but $A \cup B$ = {b,c,d} is not nano semi I-closed. That is the union of nano semi I-closed sets is not nano semi I-closed.

Theorem 6.7 : If in a nano ideal space {U, $\tau_R(X)$, I}, $L_R(X) \neq \phi$, $U_R(X)=U$ and for every ideal intersection of any two nano semi I-open sets is nano semiI-open.

Proof: U,ϕ, $L_R(X)$ and *$B_R(X)$* are the only nano are semi *I*-open sets in *U*. If A and B are nano-empty, proper nano semi *I*-open sets in U, then A=$L_R(X)$ and B=$L_R(X)$ *or* A=L_R(X) and B=L_R(X) or A=B_R(X) and B=B_R(X), If atleast one of A or B is U and in all cases, $A \cap B$ is nano semi *I*-open.

7 Nano αI-open sets :

In this segment we probed nano α*I*-open sets and pecularize their interpretation based on the intrinsic structure of nano topological space and option of ideal.

Definition 7.1 : A subset A of an ideal space {U, $\tau_R(X)$, I} is said to be nano αI-open if $A \subseteq NInt[Ncl^*[NInt(A)]$. We call a subset A of U is nano αI-closed if its complement is nano αI-open.

Theorem 7.2 : If in a nano ideal topological space {U, $\tau_R(X)$,I} and $L_R(X)$= $UR(X)$

(i) If $L_R(X)$) $\notin$ I, then ϕ ,U and those sets for which $A \supseteq L_R(X)$ are the only nano αI-open sets in U.

(ii) If $L_R(X) \in$ I, then U, ϕ , $L_R(X)$ are the only nano αI-open sets in U.

Proof: Since $L_R(X) = UR(X) = X$, the nano topology, $\tau_R(X)$ = {U, ϕ $L_R(X)$ }.

(i) If $L_R(X) \notin I$ and A = $L_R(X)$, then $NInt(A)$= $L_R(X)$. Therefore $NInt[(Ncl^*[NInt(L_R(X))]=U$. Hence A is nano α*I*-open. If $A \subset L_R(X)$, then $NInt(A)=\phi$. Therefore $NInt[(Ncl^*[NInt(L_R(X)] =\phi$ Hence A is not nano α*I* -open. If $A \supset L_R(X)$, then $NInt[(Ncl^*[NInt(L_R(X))] = U$. Hence A is nano α*I*-open. Thus ϕ, *U* and those sets for which $A \supseteq L_R(X)$ are the only nano α*I*-open sets in *U*.

(ii) If $L_R(X) \in I$ and A= $L_R(X)$, then $NInt(A)$= $L_R(X)$. Therefore $NInt[(Ncl^*[NInt(L_R(X)]$= $L_R(X)$. Hence A is nano α*I*-open. If $A \subset L_R(X)$, then $NInt(A)=\phi$. Therefore $NInt[(Ncl^*[NInt(L_R(X)] = \phi$. Hence A is not nano α*I*-open.

If $A \supset L_R(X)$, then $NInt[(Ncl^*[NInt(L_R(X)]$= $L_R(X)$. Hence

$A \subseteq NInt[(Ncl^*[NInt(A)]$. Therefore A is not nano αI-open. Thus U, ϕ and $L_R(X)$ are the only nano αI-open sets in U.

Theorem 7.3 : If in a nano ideal topological space $\{U, \tau_R(X), I\}$ *and* $L_R(X) = \phi$ $U_R(X) \neq U$

(i) If $U_R(X) \notin I$*, then* ϕ, U *and those sets for which* $A \supseteq U_R(X)$ *are the only nano* αI*-open sets in* U.

(ii) If $U_R(X) \in I$*, then* $U, \phi, U_R(X)$ *are the only nano* αI*-open sets in* U.

Theorem 7.4 : If $U_R(X) = U$ *and* $L_R(X) \neq \phi$ *in a nano topological space and for every ideal on* U *the nano* αI*-open sets in* U *are* U, ϕ, $L_R(X)$ *and* $B_R(X)$.

Proof: Since $U_R(X) = U$ and $L_R(X) \neq \phi$ the nano open sets are U, ϕ, $L_R(X)$ and $B_R(X)$. Let $A \subseteq U$ such that $A \neq \phi$. If $A = \phi$, then A is nano αI-open. Therefore, let $A \neq \phi$ when $A \subset L_R(X)$, $NInt(A) = \phi$, Since the largest nano open subset of A is ϕ and Hence $A \not\subset NInt[(Ncl^*[NInt(A)]$. Therefore A is not nano αI-open in U. when $L_R(X) \subset A$, $NInt(A) = L_R(X)$ and therefore $NInt[(Ncl^*[NInt(A)] = L_R(X) \subset A$. That is $A \not\subset NInt[(Ncl^*[NInt(A)]$. Hence A is nano αI-open in U. when $A = L_R(X)$, $NInt[(Ncl^*[NInt(A)] = L_R(X)$, Hence A is nano αI-open. Similarly it can be shown that any set $A \subset B_R(X)$ and $A \supset B_R(X)$ are not nano αI-open in U. But $A = B_R(X)$, is nano αI-open in U. If A has atleast one element, each of $L_R(X)$ and $B_R(X)$ then $NInt(A) = \phi$ and Hence A is not nano αI-open in U. Thus U, ϕ, $L_R(X)$ and $B_R(X)$ are the only nano αI-open in U.

Theorem 7.5 : Let $L_R(X)) \neq U_R(X)$, $L_R(X) \neq \phi$ $U_R(X)) \neq U$ *in a nano topological space and for every ideal other than* $U_R(X)$ *then the nano* αI*-open sets in* U *are* U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$ *and any set containinig* $U_R(X)$.

Proof: The nano topology on U is given by $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$. It is obvious A is nano αI-open when $A = L_R(X)$, $A = U_R(X)$ and $A = B_R(X)$. Let $A \subseteq U$ such $A \supset U_R(X)$. Then $NInt(A) = U_R(X)$ and therefore $NInt[(Ncl^*[NInt(A)] = U$. Hence $A \subseteq NInt[(Ncl^*[NInt(A)]$. Therefore any $A \supset U_R(X)$ is nano αI-open sets in U. If $A \subset L_R(X)$. Then $NInt(A) = \phi$ and therefore $NInt[(Ncl^*[NInt(A)] = \phi$. Therefore any $A \subset L_R(X)$ is not nano αI-open sets in U. If $A \subset B_R(X)$, then $NInt(A) =$ and therefore $NInt[(Ncl^*[NInt(A)] = \phi$. Thus any $A \subset B_R(X)$ is not nano αI-open sets in U. If $A \subset U_R(X)$ such that A is neither a subset of $L_R(X)$ nor a subset of $B_R(X)$, $NInt(A) = \phi$ and therefore $NInt[(Ncl^*[NInt(A)] = \phi$. Therefore any $A \subset U_R(X)$ is not nano αI-open sets in U. Thus U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$ and any set containinig $U_R(X)$ are nano αI-open sets in U.

Proposition 7.6 : Let $L_R(X)) \neq U_R(X)$, $L_R(X) \neq \phi$ $U_R(X) \neq U$ *in a nano topological space and if* $U_R(X) \in I$ *then the nano* αI*-open sets in* U *are* U, ϕ, $L_R(X)$, $U_R(X)$ *and* $B_R(X)$.

Proposition 7.7: If in a nano ideal space $\{U, \tau_R(X), I\}$, $L_R(X) \neq U_R(X)$ *and for every ideal* I *other than* $U_R(X)$*, then the only nano* αI*-open sets are* $U, \phi, L_R(X)$ *and* $B_R(X)$.

8 Nano RegularI-open sets :

In this section we acquaint Nano R*I*-open sets in a nano ideal space and explored their properties in terms of approximations.

Definition 8.1 : A subset A of an ideal space {U, $\tau_R(X)$ *,I} is said to be an nano regular I-open sets or nano RI-open if A= NInt(Ncl*(A)). We call a subset A of U is nano RI-closed if its complement is nano RI-open.*

Example 8.2 : Let U = {a,b,c,d} and U / R = {{a},{b,c,d}} and X = {a,c} $\subseteq$ *U. Then in the nano ideal space* $\tau_R(X)$ *= {U, ϕ, {a},{b,c,d}} with an nano ideal I = {{ϕ}, {a}}, the nano RI-open sets in U are U, ϕ, {a}, {b,c,d}.*

Theorem 8.3 : Any nano RI-open set is nano open.

Proof: If A is nano R*I*-open in *{U,* $\tau_R(X)$*, I}* then A= *NInt*(*Ncl**(*A*)). Then *NInt*(*A*)= *NIntNInt*(*Ncl**(*A*)) = *A*. That is, A is nano open in *U*.

Remark 8.4 : The converse of the above theorem is not true. For example Let U = {a,b,c,d,e} and U/R = {{a}, {,b,c},{d,e}} and X ={a,d,e}⊆U. Then in the nano ideal space $\tau_R(X)$ *={U,ϕ, {a}, {a,d,e},{d,e}} with an nano ideal I = {{ϕ}, {a}}, the nano RI-open sets in U are U, ϕ, {a}, {d, e}. Thus we note that {a,d,e} is nano open but it is not nano RI-open.*

Theorem 8.5: Let {U, $\tau_R(X)$ *,I} be a nano ideal topological space and if* $L_R(X) = U_R(X)$*, then*

$$NRI\text{ - opensets} = \begin{cases} U,\ \phi,\ L_R(X) & \text{if } L_R(X) \in I \\ U,\ \phi & \text{if } L_R(X) \notin I \end{cases}$$

Proof: If $L_R(X) = U_R(X)$, then nano open sets in *U* are *U*, ϕ , $L_R(X)$.

(i): If $L_R(X) \in I$, then $NInt(Ncl^*(L_R(X))) = U \neq L_R(X)$. Therefore $L_R(X)$ is not $NRI - open$. Thus, the only $NRI - open$ sets are *U* and ϕ.

(ii): If $L_R(X) \notin I$, then $NInt(Ncl^*(L_R(X)) = L_R(X)$. Therefore

$L_R(X)$ is $NRI - open$. Thus, the only $NRI - open$ sets are *U*, ϕ and $L_R(X)$.

Theorem 8.6 : Let {U, $\tau_R(X)$*, I} be a nano ideal topological space and if* $L_R(X) = \phi$ *and* $U_R(X) \neq U$*, then*

$$NRI\text{–opensets} = \begin{cases} U, \phi,\ U_R(X) & \text{if } U_R(X) \in I \\ U,\ \phi & \text{if } U_R(X) \notin I \end{cases}$$

Theorem 8.7 : For every ideal I and if $L_R(X) \neq \phi$ *and* $U_R(X) = U$ *the only nano RI-open sets in U are U,ϕ ,* $L_R(X)$ *and* $B_R(X)$.

Proof: $\tau_R(X) = \{U,\ \phi, L_R(X),\ B_R(X)\}$. Since $NInt(Ncl^*(L_R(X)) = L_R(X)$ and $NInt(Ncl^*(B_R(X)) = B_R(X)$. Thus *U*,$\phi$, $L_R(X)$ and $B_R(X)$ are the only nano R*I*-open sets in *U*.

Theorem 8.8 : If in a nano ideal space {U, $\tau_R(X)$ *,I},* $L_R(X) \neq U_R(X)$ *and* $U_R(X) \in I$ *then the only nano RI-open sets are U,ϕ,* $L_R(X)$*,* $U_R(X)$ *and* $B_R(X)$.

Proof: If $L_R(X) \neq U_R(X)$ then the only nano open sets are *U*, ϕ, $L_R(X)$, $U_R(X)$ and *BR*(*X*). If A= $L_R(X)$, then $NInt(Ncl^*(L_R(X)) =$

$L_R(X)$. Therefore A is nano RI-open. If A= $U_R(X)$, then $NInt(Ncl*(U_R(X))$ = $U_R(X)$. Therefore A is nano RI-open. If A= $B_R(X)$, then $NInt(Ncl*(B_R(X))$ = $B_R(X)$. Therefore A is nano RI-open. Hence the only nano RI-open sets are U, ϕ, $L_R(X)$, $U_R(X)$ and $B_R(X)$.

Theorem 8.9 : If in a nano ideal space $\{U, \tau_R(X), I\}$, $L_R(X) \neq U_R(X)$ *and for every ideal I other than* $U_R(X)$*, then the only nano RI-open sets are* U, ϕ, $L_R(X)$ *and* $B_R(X)$.

9 Comparision of nano ideal space and nano topological space in weaker forms :

In this section we have made comparisions of weaker forms of nano open sets among nano ideal space and nano topological space.

Proposition 9.1 : In a nano ideal topological space every nano semi I-open set is nano semi-open.

Proof: Let A be a nano semi$I$-open set in U, then A$\subseteq Ncl*[NInt(A)]$ $\subset [NInt(A)]* \cup [NInt(A)] \subset Ncl[NInt(A)] \cup NInt(A) = Ncl[NInt(A)]$. Thus A is nano semi-open.

Remark 9.2 : The converse of the above the theorem need not be true which can be shown by the following example.

Example 9.3 : Let U with U/R = $\{\{a\}, \{b,c\},\{d, e\}\}$ *and let* $X=\{\{a, e\}\} \subseteq U$*, then* $\tau_R(X) = \{U,\phi, \{a\}, \{a,d,e\}, \{d, e\}\}$ *with* $I = \{\phi,\{a,d,e\},\{a,d\}; \{a, e\}, \{d, e\}, \{a\}, \{d\}, \{e\}\}$ *be an nano ideal topological space, then A =* $\{a,b\}$ *is nano semi-open in U, but not nano semiI-open in U.*

Proposition 9.4: In a nano ideal space, every nano αI*-open set is nano* α*-open set.*

Proof: Let A be a nano αI-open set, then A$\subseteq NInt[Ncl*[NInt(A)]] = NInt[Ncl[NInt(A)] \cup Int(A)] \subset NInt[Ncl[NInt(A)]$. Thus A is nano αI-open.

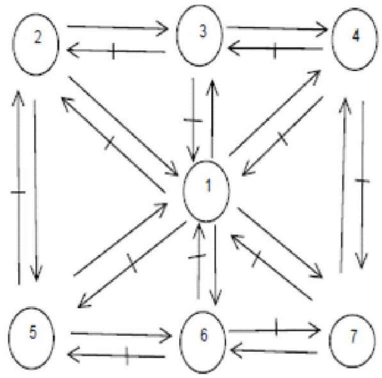
Remark 9.5 : The converse of the above theorem need not be true, which can be shown by the following example.

Example 9.6 : Let U with U/R = $\{\{a\}, \{b,c\},\{d\}\}$ *and let* $X=\{\{a,d\}\} \subseteq U$*, then* $\tau_R(X)=\{U, \phi,\{a,d\}\}$ *with* $I = \{\phi, \{a, d\}, \{a\}, \{d\}\}$ *be an nano ideal topological space, then A =* $\{a,c,d\}$ *is nano* α *-open in U, but not nano* αI*-open in U.*

Proposition 9.7 : Every nano open set of an nano ideal space is nano αI*-open.*

Proof: Let A be any nano open set,then A=$NInt(A) \subseteq NInt[NInt(A)*] \cup NInt(A) = NInt[Ncl*[NInt(A)]]$. Thus A is nano αI-open.

Remark 9.8 : In the nano ideal space several forms of weak nano open sets defined above have the following implications, which is shown in the following figure.



1.Nano open 2. Nano regular open
3. nano semi*I*-open 4.nano semi-open
5.nano R*I*-open 6.nano α-open
7.nano α*I*-open

10 Conclusion

In nano ideal topological space we can further focus into stronger form of nano open sets and various forms of continuity can also be discussed in future. We believe that the approaches we offer here will turn out to be more useful for practical applications in nano ideal space and help us to gain much more insights into the mathematical structures of nano approximations.

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