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Method of Construction of Triangular Designs

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Abstract

This paper describes a new method of construction of triangular designs. Consequently, a new series of truly self-dual triangular design is obtained.

*Keywords:*BIB design; GD design.

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1. Introduction

A triangular association scheme is defined as follows:

Let S be a set with $|s| = n$ and B a family of k- subsets called blocks of S. We assume throughout that k<v.

Consider a set of cardinality n(say), S =1, 2, …, n. The treatments of association scheme are all possible $\binom{n}{2}$ unordered pairs of elements of S. Two unordered pairs corresponding to two different treatments are first associates, if they have one element is common, otherwise second associates, e.g. (i, j) and (i, j^1) are first associates if ($j \neq j^1$). The following relations clearly holds:

$$n_1 = 2(n-2), \quad n_2 = \frac{(n-2)(n-3)}{2}$$

$$p_1 = \begin{pmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{pmatrix} = \begin{pmatrix} n-2 & n-3 \\ n-3 & \dfrac{(n-3)(n-4)}{2} \end{pmatrix}$$

$$p_2 = \begin{pmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{pmatrix} = \begin{pmatrix} 4 & 2n-8 \\ 2n-8 & \frac{(n-4)(n-5)}{2} \end{pmatrix}$$

For the definition and other combinatorial properties of triangular design refer Raghavarao[4], for the classification and analysis of triangular design refer Bose and Shimamoto[1]. Dulawat *et al.*[3] has obtained some methods of construction of nested group divisible (GD) designs.

Definition: A design D which is self-dual and whose dual D' is isomorphic to D is a truly self-dual design. A truly self dual (TSD) design is always a self-dual(SD) design but a SD design may not be a TSD design.

2. Construction:

In this section, we describe the method how truly self-dual triangular designs are obtained. In particular, we give two triangular designs which are truly self dual.

Theorem: A series of two associate class triangular partially balanced incomplete block (PBIB) designs with parameters:

$$v = \frac{n(n-1)}{2},\ b = \frac{n(n-1)}{2},\ r = \frac{(n-2)(n-3)}{2},\ k = \frac{(n-2)(n-3)}{2}$$

$$\lambda_1 = \frac{(n-2)(n-3)}{4},\ \lambda_2 = \frac{(n-2)(n-3)-2(n-1)}{2},\quad n_1 = 2(n-2) \tag{1.1}$$

$$n_2 = \frac{(n-2)(n-3)}{2}$$

Always existswhen $n \geq 6$ and is truly self-dual.

Proof: Suppose S be the set of *n* positive integers numbered from 1,2,…, n. We identify 2-sets of S as treatments and (n-2)-sets of *S* as blocks. A block corresponding to (n-2)-set of S consists of all 2-sets formed with the (n-2)-set. Thus, we get the design with parameters given in (1.1), which can be easily verified.

Now by introducing the isomorphism

$$(i_1 i_2 \ldots\ldots) \leftrightarrow (i_{2+1} \ldots\ldots i_n)$$

where $i_1 \ldots\ldots i_2 \neq i_{2+1} \ldots\ldots i_n;\ i = 1, 2, \ldots, n,$ between treatments and blocks and noting that if n<2 (n-2) the association scheme is with two associate classes. It can also be verified that the design with parameters (1.1) is truly self-dual.

As a particular case of (1.1) for n =7, we get a triangular PBIB design with parameters:

$$v = b = 21,\ r = k = 10,\ \lambda_1 = 5,\ \lambda_2 = 4;\ n_1 = 10,\ n_2 = 10$$

We find that the triangular design T_{95} in Clatworthy[2] and the triangular design obtained by us with parameters (1.1) for n=7 are isomorphic. Therefore, two associate class triangular design T_{95} is in fact truly self-dual. But T_{95} in Clatworthy[2] is marked as only self-dual.

Triangular design T_{95} obtained by us which is truly self-dual whose blocks are written in the form of columns:

1	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**3**	**3**	**3**	**4**	**4**	**5**
3	**3**	**2**	**2**	**2**	**2**	**2**	**3**	**4**	**5**	**6**	**3**	**4**	**5**	**6**	**4**	**5**	**6**	**5**	**6**	**6**
4	**4**	**4**	**3**	**3**	**3**	**8**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**8**	**8**	**8**	**9**	**9**	**10**
5	**5**	**5**	**6**	**4**	**4**	**9**	**9**	**8**	**8**	**8**	**8**	**9**	**10**	**11**	**9**	**10**	**11**	**10**	**11**	**11**
6	**6**	**6**	**9**	**6**	**5**	**10**	**10**	**10**	**9**	**9**	**13**	**12**	**12**	**12**	**12**	**12**	**12**	**13**	**13**	**14**
7	**7**	**8**	**13**	**10**	**11**	**11**	**11**	**11**	**11**	**10**	**14**	**14**	**13**	**13**	**13**	**14**	**15**	**14**	**15**	**15**
12	**8**	**12**	**5**	**14**	**15**	**12**	**12**	**13**	**14**	**15**	**15**	**15**	**15**	**14**	**17**	**16**	**16**	**16**	**16**	**17**
13	**9**	**16**	**16**	**17**	**18**	**13**	**16**	**16**	**17**	**18**	**16**	**16**	**17**	**18**	**18**	**18**	**17**	**17**	**18**	**18**
14	**10**	**17**	**19**	**20**	**19**	**14**	**17**	**19**	**20**	**19**	**17**	**19**	**20**	**19**	**19**	**20**	**19**	**19**	**20**	**19**
15	**11**	**18**	**21**	**21**	**20**	**15**	**18**	**21**	**21**	**20**	**18**	**21**	**21**	**20**	**21**	**21**	**20**	**20**	**21**	**21**

Remark: Triangular Design T_{94} in Clatworthy[2] is marked as self-dual whereas, we are reporting truly self-dual with parameters:

$$v = b = 21,\ r = k = 10,\ \lambda_1 = 6,\ \lambda_2 = 3;\ n_1 = 10,\ n_2 = 10$$

The solution of T_{94} obtained by us is presented below in which blocks are written in the form of columns:

1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**3**	**7**	**7**	**7**	**7**	**8**	**12**
2	**2**	**2**	**2**	**2**	**2**	**3**	**3**	**3**	**4**	**3**	**3**	**3**	**4**	**4**	**8**	**8**	**8**	**9**	**9**	**13**
3	**3**	**3**	**4**	**4**	**5**	**4**	**4**	**5**	**5**	**4**	**4**	**5**	**5**	**5**	**9**	**9**	**10**	**10**	**10**	**14**
4	**5**	**6**	**5**	**6**	**6**	**5**	**6**	**6**	**6**	**5**	**6**	**6**	**6**	**6**	**10**	**11**	**11**	**11**	**11**	**15**
7	**7**	**7**	**7**	**7**	**7**	**8**	**8**	**8**	**9**	**12**	**12**	**12**	**13**	**16**	**12**	**12**	**12**	**13**	**16**	**16**
8	**8**	**8**	**9**	**9**	**10**	**9**	**9**	**10**	**10**	**13**	**13**	**14**	**14**	**17**	**13**	**13**	**14**	**14**	**17**	**17**
9	**10**	**11**	**10**	**11**	**11**	**10**	**11**	**11**	**11**	**14**	**15**	**15**	**15**	**18**	**14**	**15**	**15**	**15**	**18**	**18**
12	**12**	**12**	**13**	**13**	**14**	**16**	**16**	**17**	**19**	**16**	**16**	**17**	**19**	**19**	**16**	**16**	**17**	**19**	**19**	**19**
13	**14**	**15**	**14**	**15**	**15**	**17**	**18**	**18**	**20**	**17**	**18**	**18**	**20**	**20**	**17**	**18**	**18**	**20**	**20**	**20**
16	**17**	**18**	**19**	**20**	**21**	**19**	**20**	**21**	**21**	**19**	**20**	**21**	**21**	**21**	**19**	**20**	**21**	**21**	**21**	**21**

References

1. Bose, R.C. and Shimamoto, T., *Classification and analysis of partially balanced incomplete block designs with two associate classes, Jr. Amer. Statist. Assoc. 47,* 151-184 (1952).
2. Clatworthy, W.H., *Tables of two - associate class partially balanced designs, NBS Applied Mathematics Series, 63, U.S. Department of Commerce, National Bureau of Standards, Washington, D.C.* (1973).
3. Dulawat, M.S., Mehta, A., Agarwal, S. and Rathore, M.S., *Some methods of construction of nested group divisible designs, Jr. Rajasthan Academy Physical Sciences 11 (2),* 179-185 (2012).
4. Raghavarao, D., *Constructions and combinational problems in design of experiments, Dover, New York.* (1988).