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Lrs Bianchi Type-II Cosmological Model with Bulk Viscosity and Dust Distribution in C-Field Theory

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Abstract

We have investigated LRS Bianchi type-II cosmological model with bulk viscosity and dust distribution in C-field theory. We assume that the matter content of the universe is in the form of dust which leads to $p = 0$. To get the deterministic model of the universe we assumed $A = B^n$, where A and B are metric potentials. We find that the creation field (C) increase with time which matches with the result of H.N. theory. The physical and geometrical aspects of the model are also discussed.

Key words : LRS Bianchi type-II, C-field, Dust, Bulk viscosity, Cosmology

1. Introduction

Homogeneous and anisotropic cosmological model have been studied in the frame work of general relativity in the search of realistic picture of the universe in its early stage. In this paper we confine our study within the scope of a Bianchi type-II space time, which has recently been studied by a number of authors. Pant and Oli[15] studied two fluid Bianchi type-II cosmological models. Bianchi type-II cosmological model with constant deceleration parameter considered by Singh and Kumar[24]. Pradhan *et al*.[18] studied the LRS Bianchi type-II cosmological models in presence of massive cosmic string and varying cosmological constant. Panov *et al*.[16] studied the Bianchi type II cosmological model of the universe's evolution. Bianchi type-II model play an important role in current model cosmology, for simplification and description of the large scale behaviour of the actual universe. Assco and Sol[2] emphasized the importance of Bianchi type-II universe. Bali[4] studied Aspects of Inflation in Spatially Homogenous and Anisotropic Bianchi type –I Spacetime with Exponential potential.

Bali and Anjali[3] has investigated Bianchi type-I magnetized string cosmological model in general relativity. Kriori *et al*.[12] and Chakraborty and Nandi[9] have investigated cosmological models for Bianchi type-II, VII space times. Rao *et al*.,[19] studied exact Bianchi type-II, VIII and IX string cosmological model in Saez-Ballester theory of gravitation. Roy and Banerjee[20] dealt with LRS cosmological models of Bianchi type-II representing clouds of geometrical as well as massive strings. Tyagi and Keerti[24] investigated the Bianchi type-II bulk viscous string cosmological models in general relativity.

In the early universe, all the investigation dealing with physical process use a model of the universe, usually called a big-bang model. The big-bang model based on Einstein field equation successfully explains the three important observation in Astronomy viz. The phenomena of expanding universe, Primordial nucleosynthesis and the observed isotropy of the cosmic background radiation.

The astronomical observation in the late eighties has revealed that the predications of FRW type models do not always meet with our requirements as we believe earlier, the big bang model is known to have the following short comings :

(i) The model has singularity in the past and possibly one in future.

(ii) The conservation of energy is violated in the big-band model.

(iii) The big-band models based on reasonable equations of state lead to a very small particle horizon in the early epochs of the universe. This fact gives ries to the 'Horizon problem'.

(iv) No consistent scenario exists within the frame work of big-band model that explains the origin, evaluation and characteristic of structure in the universe of small scales.

(v) Flatness problem.

Thus alternative theories were proposed time to time. The most well known theory is the 'steady state theory' by Bondi and Gold. In this theory the universe does not have any singular beginning nor an end on the cosmic time scale to maintain constancy of matter density, they envisaged a very slow but continuous creation of matter in contrast to the irrespective certain at $t = 0$ of the standard FRW model. However, if suffers the serious disqualification for not giving any physical justification in the form of any dynamical theory for continuous creation of matter, also the principle energy conservation was scarifies in this formulism.

To remove this problem Hoyle and Narlikar[11] adopted a field theoretic approach introducing a massless and chargeless scalar field in Einstein Hilbert action to account for creation of matter. In C-field theory, there is no big-bang type singularity as in the steady state theory of Bondi and Gold[8]. Narlikar and Padmanabhan[14] have investigated the solution of Einstein's field equation which admit radiation and negative energy massless scalar C-field as source. Bali and Tikekar[7] have investigated C-field cosmological model for dust distribution in flat FRW model with variable gravitation constant.

Bali and Kumawat[5] have investigated C-field cosmological model with variable G in FRW space time. Bali and Saraf[6] studied C-field cosmological model for dust distribution with varying Λ in FRW space time, to get deterministic model satisfying conservation equation $T^{j}_{i;j} = 0$ they have assumed $\Lambda = \frac{1}{R^2}$ where R is the scale factor. Saraf [22] studied Bianchi type I cosmological model for dust distribution with variable G and Lambda.

Saraf [21] studied a cosmological model of radiation dominated phase with dark energy in C- field cosmology. Chatterjee and Bannerjee[10] have studied C-field cosmology in higher dimensions. Singh and Chaubey[23] have investigated Bianchi type I, III, V, VI and Kantowski sach universe in creation field cosmology. Adhav *et al*.,[1] have obtained Kasner and Auxially symmetric universe in C-field theory of gravitation. Tyagi and Singh[26] have studied a cosmological model for barotropic fluid distribution in C-field cosmology with varying cosmological constant (Λ) in Bianchi type-III space time. LRS Bianchi type-V perfect fluid cosmological model in C-field theory with variable Λ have studied by Tyagi and Singh[27]. Parikh and Tyagi[17] have studied time dependent Λ in Bianchi type-IX space time with barotropic perfect fluid in C-field. Mehta and Chundawat[13] have studied LRS Bianchi type II cosmological model with barotropic perfect fluid in C –Field theory with time dependent term.

In this paper, we have investigated LRS Bianchi type-II cosmological model with bulk viscosity and dust distribution in C-field theory we assume that the matter content of the universe is in the form of dust, which leads to p = 0. To get the deterministic model of the universe we have assumed $A = B^n$, where A and B are metric potentials. We find that the creation field (C) increase with time which matches with the result of H.N. theory. The physical and geometrical aspects of the model are also discussed.

2. The Metric and Field Equations :

We consider LRS Bianchi type-II space time in form of

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy - xdz)^2 \tag{1}$$

where the metric potential A and B are function of t alone and $\sqrt{-g} = A^2B$

Hoyle and Narlikar [11] modified the Einstein field equation by introducing C-field as :

$$R^{j}_{i} - \frac{1}{2}Rg^{j}_{i} = -8\pi G\left[\underset{(m)}{T^{j}_{i}} + \underset{(c)}{T^{j}_{i}}\right] \tag{2}$$

The energy momentum tensor $\underset{(m)}{T^{j}_{i}}$ for bulk viscous fluid distribution and for creation field $\underset{(c)}{T^{j}_{i}}$ are given by

$$\underset{(m)}{T^{j}_{i}} = (\bar{p} + \rho)\, v_i v^j + \bar{p} g^{j}_{i} \quad \text{where} \quad \bar{p} = p - \xi\theta \tag{3}$$

$$\underset{(c)}{T^{j}_{i}} = -f\left(c_i c^j - \frac{1}{2} g^{j}_{i} c_\alpha c^\alpha\right) \tag{4}$$

where f > 0 is coupling constant between the matter and creation field and $C_i = \frac{dC}{dx^i}$.

The co-moving coordinate are chosen such that $v^i = (0, 0, 0, 1)$. The non-vanishing components of energy momentum tensor for matter are given by

$$\underset{(m)}{T_1^1} = \underset{(m)}{T_2^2} = \underset{(m)}{T_3^3} = -\xi\theta \text{ and } \underset{(m)}{T_4^4} = -\rho \tag{5}$$

The non-vanishing components of energy momentum tensor for creation field are given by

$$\underset{(c)}{T_1^1} = \underset{(c)}{T_2^2} = \underset{(c)}{T_3^3} = -\frac{1}{2}f\dot{C}^2 \text{ and } \underset{(c)}{T_4^4} = \frac{1}{2}f\dot{C}^2 \tag{6}$$

Hence the Einstein field equation (2) for the metric (1) and energy momentum tensor (5) and (6) takes the form

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B^2}{4A^4} = 8\pi G\left[\xi\theta + \frac{1}{2}f\dot{C}^2\right] \tag{7}$$

$$\frac{2A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{3B^2}{4A^4} = 8\pi G\left[\xi\theta + \frac{1}{2}f\dot{C}^2\right] \tag{8}$$

$$\frac{A_4^2}{A^2} + \frac{2A_4 B_4}{AB} - \frac{B^2}{4A^4} = 8\pi G\left[\rho - \frac{1}{2}f\dot{C}^2\right] \tag{9}$$

The suffix 4 by the symbols A and B denotes differentiation with respect to 't'.

2. Solution of Field Equations :

The conservation equation

$$[8\pi G T_i^j]_{;j} = 0 \tag{10}$$

which leads to

$$8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + \{(\rho - \overline{p}) - f\dot{C}^2\}\left(\frac{2A_4}{A} + \frac{B_4}{B}\right)\right] = 0 \tag{11}$$

following Hoyle and Narlikar [11], the source equation of C-field $C_{;i}^{i} = \frac{n}{f}$ leads to C = t for large r, thus $\dot{C} = 1$.

To get deterministic solution of equation (7) to (9), we assume a condition between the metric potential as

$$A = B^n \tag{12}$$

Using equation (12), equation (7) to (9) becomes

$$(n+1)\frac{B_{44}}{B}+\frac{n^2B_4^2}{B^2}+\frac{B^2}{4B^{4n}}=8\pi G\left[\xi\theta+\frac{1}{2}f\dot{C}^2\right] \tag{13}$$

$$2n\frac{B_{44}}{B}+(3n^2-2n)\frac{B_4^2}{B^2}-\frac{3B^2}{4B^{4n}}=8\pi G\left[\xi\theta+\frac{1}{2}f\dot{C}^2\right] \tag{14}$$

$$(2n+n^2)\frac{B_4^2}{B^2}-\frac{B^2}{4B^{4n}}=8\pi G\left[\rho-\frac{1}{2}f\dot{C}^2\right] \tag{15}$$

Equation (13) and (14) together leads to

$$(n-1)\frac{B_{44}}{B}+2n(n-1)\frac{B_4^2}{B^2}=\frac{B^2}{B^{4n}} \qquad \ldots(16)$$

Equation (16) leads to

$$2B_{44}+\frac{4n}{B}B_4^2=\frac{2}{(n-1)}\frac{B^3}{B^{4n}} \tag{17}$$

Let $B_4 = f(B)$ which leads to $B_{44}=ff'$

Equation (17) leads to

$$\frac{df^2}{dB}+\frac{4n}{B}f^2=\frac{2}{(n-1)}B^{(3-4n)} \tag{18}$$

Equation (18) leads to

$$f^2=\frac{1}{2(n-1)}B^{4(1-n)} \tag{19}$$

The constant of integration has been taken zero for simplicity.

Equation (19) leads to

$$\frac{dB}{B^{2(1-n)}}=\sqrt{\frac{1}{2(n-1)}}dt \tag{20}$$

Equation (20) leads to

$$B=(Kt+\alpha)^{\frac{1}{2n-1}} \tag{21}$$

where $K=\sqrt{\frac{(2n-1)^2}{2(n-1)}}$ and $\alpha = M(2n-1)$

Thus the metric (1) after using equation (21) leads to

$$ds^2=-dt^2+(Kt+\alpha)^{\frac{2n}{2n-1}}(dx^2+dz^2)+(Kt+\alpha)^{\frac{2}{2n-1}}(dy-xdz)^2 \tag{22}$$

Equation (11) and equation (12) leads to

$$8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + \{(\rho+\xi\theta) - f\dot{C}^2\}(2n+1)\left(\frac{B_4}{B}\right)\right] = 0 \quad (23)$$

Equation (23) leads to

$$\frac{d\dot{C}^2}{dt} + \frac{(2n+1)}{(2n-1)}\frac{2K}{(Kt+\alpha)}\dot{C}^2$$

$$= \frac{1}{4\pi Gf}\left[\frac{(2n+n)^2}{(2n-1)^2}\left(\frac{-2K^3}{(Kt+\alpha)^3}\right) + \frac{K}{2}\frac{1}{(Kt+\alpha)^3}\right] +$$

$$\frac{1}{4\pi Gf}\left[\frac{(2n+n^2)}{(2n-1)}\frac{K^2}{(Kt+\alpha)^2} - \frac{1}{4}\frac{1}{(Kt+\alpha)^2} + 1\right]$$

$$\left\{\frac{K(2n+1)}{(2n-1)}\frac{1}{(Kt+\alpha)}\right\} + \frac{2}{f}\xi\theta\frac{K(2n+1)}{(2n-1)(Kt+\alpha)} \quad (24)$$

Equation (24) for $n = \frac{3}{2}$ and $\xi\theta = \frac{f}{2}$ leads to

$$\frac{d\dot{C}^2}{dt} + \frac{4K}{(Kt+\alpha)}\dot{C}^2 = \frac{4K}{(Kt+\alpha)} \quad (25)$$

Equation (25) leads to

$$\dot{C}^2(Kt+\alpha)^2 = (Kt+\alpha)^2 + Q \quad (26)$$

For simplicity, we assume that integral constant Q = 0 which leads to

$$\dot{C}^2 = 1 \quad (27)$$

Thus, we have $\dot{C} = 1$ or C = t (28)

We find C = t , which agrees with the value used in the source equation. Thus creation field C is proportional to time t.

4. Some Physical And Geometrical Features :

The homogeneous mass density (ρ), the creation field (C), spatial volume (R^3), the deceleration parameter (q), shear tensor (σ) and expansion (θ) of the model (21) are given by

$$\rho = \frac{1}{8\pi G}\left[\frac{K^2(2n+n^2)}{(2n-1)^2} - \frac{1}{4}\right]\frac{1}{(Kt+\alpha)^2} + \frac{f}{2} \quad (29)$$

$$C = t \quad (30)$$

$$R^2 = (Kt+\alpha)^{\frac{n+2}{(2n-1)}} \tag{31}$$

$$q = \frac{5(n-1)}{(n+2)} \tag{32}$$

$$\theta = \frac{(n+2)K}{(2n-1)}\frac{1}{(Kt+\alpha)} \tag{33}$$

$$\sigma^2 = \frac{1}{3}\frac{(n+2)^2K^2}{(2n-1)^2}\frac{1}{(Kt+\alpha)^2} \tag{34}$$

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} = \text{Constant} \tag{35}$$

5. Conclusion

The scale factor R increase with time, since deceleration parameter $q > 0$, hence the model (22) represent decelerating universe. The density decrease as time increase. The model (22) passes through a singular state at $t = -\alpha / K$, this is explained as Creation exists all the time, so there is a big crunch between $-\alpha / K$ to ∞ and Creation is going on front $t = -\alpha / K$ to ∞. During this period the model exist. Since $t \to \infty, \frac{\sigma}{\theta}$ is constant, therefore model (22) does not approaches to isotropy in late time. Since $\theta \neq \infty$ at $t = 0$, hence model (22) is free from initial singularity.

The coordinate distance γ_H to the horizon is the maximum distance a null ray could have travelled at time t starting from infinite past i.e.

$$\gamma_H = \int_{-\infty}^{t} \frac{dt}{R^3(t)}$$

We could extent the proper time t to in the pas because of non-singular nature of space time, thus

$$\gamma_H(t) = \int_0^t \frac{dt}{R^3(t)} = \int_0^t \frac{dt}{(Kt+\alpha)^{\frac{n+2}{(2n-1)}}}$$

The integral of diverge at lower limit shows that the model is free from event horizon.

Scope and Applications: Investigation of Creation field cosmological models creates more interest in the study because it solves the outstanding problems of Big Bang cosmology like singularity, horizon and flatness problems. These models also represent inflationary scenario which shows accelerating behavior of universe and applicable in isotropization of universe.

Abbreviations

1. LRS:- Locally Rotationally Symmetric

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