ISSN 2231-346X (Print)

JUSPS-A Vol. 28(7), 370-382 (2016). Periodicity-Monthly

Section A

ISSN 2319-8044 (Online)

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES

An International Open Free Access Peer Reviewed Research Journal of Mathematics

website:- www.ultrascientist.org

Estd. 1989

Level operators Over Bipolar Intuitionistic M- Fuzzy Group and Anti M- Fuzzy Group

K. GUNASEKARAN*, S. NANDAKUMAR** and D. GUNASEELAN*

*Ramanujan Research Centre, PG and Research Department of Mathematics, Government Arts College (Autonomous), Kumbakonam-612001, Tamilnadu, India

E-Mail of Corresponding Author :-gngseelan86@gmail.com

** PG and Research Department of Mathematics, Government Arts college Ariyalur,

http://dx.doi.org/10.22147/jusps-A/280707

Acceptance Date 18th Nov., 2016, Online Publication Date 2nd Dec., 2016

Abstract

In this paper the concept of a Bipolar intuitionistic M fuzzy group and anti M fuzzy group is a new algebraic structure of a bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group are defined and related operators and level operators are investigated. The purpose of the study is to implement the fuzzy set theory and group theory of bipolar intuitionistic M fuzzy group and anti M fuzzy group. The relation between operation of level operators of bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group are established.

Key words: Bipolar fuzzy set, Bipolar M fuzzy group, bipolar anti M fuzzy group, bipolar intuitionistic M fuzzy group, bipolar intuitionistic anti M fuzzy group.

2010 Mathematics subject Classification:03E72,20E07,08A72, 03F55, 20N25.

1. Introduction

The concept of fuzzy sets was initiated by L.A. Zadeh[15] then it has becomes a vigorous area of research in engineering, medical science, graph theory. Rosenfeld[14] gave the idea of fuzzy subgroups. The author W.R. Zhang[16] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. In case of bipolar valued fuzzy sets membership degree is enlarged from the interval [0,1] to [-1,1]. In a bipolar valued fuzzy sets, the membership degree '0' means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements satisfy the property and the membership degree [-1,0) indicates that elements satisfy the implicit counter property. Atanassov. K.T[1-7] introduced intuitionistic fuzzy sets and new operations defined over intuitionistic fuzzy sets. He was investigated operators over interval valued intuitionistic fuzzy sets and level operators on intuitionistic fuzzy sets. He introduced necessity and possibility operators on intuitionistic fuzzy sets are defined in some logics. This fact that intuitionistic fuzzy sets are proper extension of ordinary fuzzy sets. R. Muthuraj[9,10] introduced the concept of bipolar M fuzzy group and bipolar anti M fuzzy group. M. Palanivelrajan and S. Nandakumar[11,12,13] introduced some operations of intuitionistic fuzzy primary and semi primary ideal and operation on operators and level operators of intuitionistic fuzzy primary and fuzzy semi primary ideal. We discuss some of its properties and some operations on operator and level operator of bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group are established.

2. Preliminaries :

In this section the fundamental definitions that will be used in the sequel. Throughout this paper, $G=(G,*)$ is a finite groups, e is the identity element of G, and xy mean x*y.

Definition.2.1.[1] Let A be an bipolar intuitionistic fuzzy set then the level operator **!** is defined by

$$!(A)^{+}=\left\{x,\max\left(\frac{1}{2},\mu_A^{+}(x)\right),\ \min\left(\frac{1}{2},\upsilon_A^{+}(x)\right)\ \Big/\ x\in E\right\}=L_1^{+}\ \ \textit{and}$$

$$!(A)^{-}=\left\{x,\min\left(\frac{-1}{2},\mu_A^{-}(x)\right),\max\left(\frac{-1}{2},\upsilon_A^{-}(x)\right)\Big/x\in E\right\}=L_1^{-}$$

Definition.2.2.[1] Let A be an bipolar intuitionistic fuzzy set then the level operator **?** is defined by

$$?(A)^{+}=\left\{x,\min\left(\frac{1}{2},\mu_A^{+}(x)\right),\ \max\left(\frac{1}{2},\upsilon_A^{+}(x)\right)\ \Big/\ x\in E\right\}=L_2^{+}\ \ \textit{and}$$

$$?(A)^{-}=\left\{x,\max\left(\frac{-1}{2},\mu_A^{-}(x)\right),\min\left(\frac{-1}{2},\upsilon_A^{-}(x)\right)\Big/x\in E\right\}=L_2^{-}$$

Definition 2.3.[4] Let A be an bipolar intuitionistic fuzzy sets of E then the necessity operator □ is defined by

$$\Box A^{+}=\left\{x,\mu_A^{+}(x),\ 1-\mu_A^{+}(x)\ \Big/\ x\in E\right\}\ \textit{and}$$

$$\Box A^{-}=\left\{x,\mu_A^{-}(x),\ -1-\mu_A^{-}(x)\Big/x\in E\right\}$$

Definition 2.4.[4] Let A be an bipolar intuitionistic fuzzy sets of E then the possibility operator ◊ is defined by

$$\Diamond A^{+}=\left\{x,1-\upsilon_A^{+}(x),\upsilon_A^{+}(x)\Big/x\in E\right\}\ \textit{and}$$

$$\Diamond A^{-}=\left\{x,-1-\upsilon_A^{-}(x),\upsilon_A^{-}(x)\Big/x\in E\right\}$$

Definition 2.5.[10] Let G be an M group and A be a bipolar intuitionistic fuzzy subgroup of G then A is called a bipolar intuitionistic M fuzzy group of G, if for all $x\in G$ and $m\in M$ then

$i)\ \mu_A^{+}(mx)\geq\mu_A^{+}(x)\ \textit{and}\ \upsilon_A^{+}(mx)\leq\upsilon_A^{+}(x)$

$ii)\ \mu_A^{-}(mx)\leq\mu_A^{-}(x)\ \textit{and}\ \upsilon_A^{-}(mx)\geq\upsilon_A^{-}(x)$

Definition 2.6.[10] Let G be an M group and A be a bipolar intuitionistic anti fuzzy subgroup of G then A is called a bipolar intuitionistic anti M fuzzy group of G, if for all $x\in G$ and $m\in M$ then

$i)\ \mu_A^{+}(mx)\leq\mu_A^{+}(x)\ \textit{and}\ \upsilon_A^{+}(mx)\geq\upsilon_A^{+}(x)$

$ii)\ \mu_A^{-}(mx)\geq\mu_A^{-}(x)\ \textit{and}\ \upsilon_A^{-}(mx)\leq\upsilon_A^{-}(x)$

Theorem 2.7. If A is an bipolar intuitionistic M fuzzy group of G then !(A) is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x,m\in G\ \textit{implies}\ x,m\in A$

Consider $\mu_{!(A)}^{+}(mx)=\max\left(\frac{1}{2},\mu_A^{+}(mx)\right)\geq\max\left(\frac{1}{2},\mu_A^{+}(x)\right)=\mu_{!A}^{+}(x)$

Therefore $\mu^{+}_{!(A)}(mx) \geq \mu^{+}_{!A}(x)$ *for some* $m \in M$ *and* $x \in G$

Consider $\upsilon^{+}_{!(A)}(mx) = \min\left(\frac{1}{2}, \upsilon^{+}_{A}(mx)\right) \leq \min\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right) = \upsilon^{+}_{!A}(x)$

Therefore $\upsilon^{+}_{!(A)}(mx) \leq \upsilon^{+}_{!A}(x)$ *for some* $m \in M$ *and* $x \in G$

Consider $\mu^{-}_{!(A)}(mx) = \min\left(\frac{-1}{2}, \mu^{-}_{A}(mx)\right) \leq \min\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right) = \mu^{-}_{!A}(x)$

Therefore $\mu^{-}_{!(A)}(mx) \leq \mu^{-}_{!A}(x)$ *for some* $m \in M$ *and* $x \in G$

Consider $\upsilon^{-}_{!(A)}(mx) = \max\left(\frac{-1}{2}, \upsilon^{-}_{A}(mx)\right) \geq \max\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right) = \upsilon^{-}_{!A}(x)$

Therefore $\upsilon^{-}_{!(A)}(mx) \geq \upsilon^{-}_{!A}(x)$ *for some* $m \in M$ *and* $x \in G$

Therefore !(A) is an bipolar intuitionistic M fuzzy group of G.

Theorem 2.8 If A and B are bipolar intuitionistic M fuzzy group of G then $!(A \cap B) = !(A) \cap !(B)$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G$ *implies* $x, m \in A$ *and* $x, m \in B$

Consider $\mu^{+}_{!(A\cap B)}(mx) = \max\left(\frac{1}{2}, \mu^{+}_{A\cap B}(mx)\right) \geq \max\left(\frac{1}{2}, \min(\mu^{+}_{A}(x), \mu^{+}_{B}(x))\right)$

$$= \min\left(\max\left(\frac{1}{2}, \mu^{+}_{A}(x)\right), \max\left(\frac{1}{2}, \mu^{+}_{B}(x)\right)\right) = \min\left(\mu^{+}_{!A}(x), \mu^{+}_{!B}(x)\right)$$

$$= \mu^{+}_{!(A)\cap !(B)}(x)$$

Therefore $\mu^{+}_{!(A\cap B)}(mx) \geq \mu^{+}_{!(A)\cap !(B)}(x)$

Consider $\upsilon^{+}_{!(A\cap B)}(mx) = \min\left(\frac{1}{2}, \upsilon^{+}_{A\cap B}(mx)\right) \leq \min\left(\frac{1}{2}, \max(\upsilon^{+}_{A}(x), \upsilon^{+}_{B}(x))\right)$

$$= \max\left(\min\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right), \min\left(\frac{1}{2}, \upsilon^{+}_{B}(x)\right)\right) = \max\left(\upsilon^{+}_{!A}(x), \upsilon^{+}_{!B}(x)\right)$$

$$= \upsilon^{+}_{!(A)\cap !(B)}(x)$$

Therefore $\upsilon^{+}_{!(A\cap B)}(mx) \leq \upsilon^{+}_{!(A)\cap !(B)}(x)$

Consider $\mu^{-}_{!(A\cap B)}(mx) = \min\left(\frac{-1}{2}, \mu^{-}_{A\cap B}(mx)\right) \leq \min\left(\frac{-1}{2}, \max(\mu^{-}_{A}(x), \mu^{-}_{B}(x))\right)$

$$= \max\left(\min\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right), \min\left(\frac{-1}{2}, \mu^{-}_{B}(x)\right)\right) = \max\left(\mu^{-}_{!A}(x), \mu^{-}_{!B}(x)\right)$$

$$= \mu^{-}_{!(A)\cap !(B)}(x)$$

Therefore $\mu^{-}_{!(A\cap B)}(mx) \leq \mu^{+}_{!(A)\cap !(B)}(x)$

Consider $\upsilon^{-}_{!(A\cap B)}(mx) = \max\left(\frac{-1}{2}, \upsilon^{-}_{A\cap B}(mx)\right) \geq \max\left(\frac{-1}{2}, \min(\upsilon^{-}_{A}(x), \upsilon^{-}_{B}(x))\right)$

$$= \min\left(\max\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right), \max\left(\frac{-1}{2}, \upsilon^{-}_{B}(x)\right)\right) = \min\left(\upsilon^{-}_{!A}(x), \upsilon^{-}_{!B}(x)\right)$$

$$= \upsilon^{-}_{!(A)\cap !(B)}(x)$$

Therefore $\upsilon^{-}_{!(A\cap B)}(mx) \geq \upsilon^{-}_{!(A)\cap !(B)}(x)$

Therefore $!(A \cap B) = !(A) \cap !(B)$ is an bipolar intuitionistic M fuzzy group of G.

Theorem 2.9. If A is an bipolar intuitionistic M fuzzy group of G then $?(A)$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G \text{ implies } x, m \in A$

$$\text{Consider} \quad \mu^+_{?(A)}(mx) = \min\left(\frac{1}{2}, \mu^+_A(mx)\right) \geq \min\left(\frac{1}{2}, \mu^+_A(x)\right) = \mu^+_{?(A)}(x)$$

$$\text{Therefore} \quad \mu^+_{?(A)}(mx) \geq \mu^+_{?(A)}(x) \text{ for some } m \in M \text{ and } x \in G$$

$$\text{Consider} \quad \upsilon^+_{?(A)}(mx) = \max\left(\frac{1}{2}, \upsilon^+_A(mx)\right) \leq \max\left(\frac{1}{2}, \upsilon^+_A(x)\right) = \upsilon^+_{?(A)}(x)$$

$$\text{Therefore} \quad \upsilon^+_{?(A)}(mx) \leq \upsilon^+_{?(A)}(x) \text{ for some } m \in M \text{ and } x \in G$$

$$\text{Consider} \quad \mu^-_{?(A)}(mx) = \max\left(\frac{-1}{2}, \mu^-_A(mx)\right) \leq \max\left(\frac{-1}{2}, \mu^-_A(x)\right) = \mu^-_{?(A)}(x)$$

$$\text{Therefore} \quad \mu^-_{?(A)}(mx) \leq \mu^-_{?(A)}(x) \text{ for some } m \in M \text{ and } x \in G$$

$$\text{Consider} \quad \upsilon^-_{?(A)}(mx) = \min\left(\frac{-1}{2}, \upsilon^-_A(mx)\right) \geq \min\left(\frac{-1}{2}, \upsilon^-_A(x)\right) = \upsilon^-_{?(A)}(x)$$

$$\text{Therefore} \quad \upsilon^-_{?(A)}(mx) \geq \upsilon^-_{?(A)}(x) \text{ for some } m \in M \text{ and } x \in G$$

Therefore $?(A)$ is an bipolar intuitionistic M fuzzy group of G.

Theorem 2.10. If A and B are bipolar intuitionistic M fuzzy group of G then $?(A \cap B) = ?(A) \cap ?(B)$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G \text{ implies } x, m \in A \text{ and } x, m \in B$

$$\text{Consider} \quad \mu^+_{?(A\cap B)}(mx) = \min\left(\frac{1}{2}, \mu^+_{A\cap B}(mx)\right) \geq \min\left(\min\left(\frac{1}{2}, \mu^+_A(x)\right), \min\left(\frac{1}{2}, \mu^+_B(x)\right)\right)$$

$$= \min\left(\mu^+_{?A}(x), \mu^+_{?B}(x)\right) = \mu^+_{?(A)\cap ?(B)}(x)$$

$$\text{Therefore} \quad \mu^+_{?(A\cap B)}(mx) \geq \mu^+_{?(A)\cap ?(B)}(x)$$

$$\text{Consider} \quad \upsilon^+_{?(A\cap B)}(mx) = \max\left(\frac{1}{2}, \upsilon^+_{A\cap B}(mx)\right) \leq \max\left(\max\left(\frac{1}{2}, \upsilon^+_A(x)\right), \max\left(\frac{1}{2}, \upsilon^+_B(x)\right)\right)$$

$$= \max\left(\upsilon^+_{?A}(x), \upsilon^+_{?B}(x)\right) = \upsilon^+_{?(A)\cap ?(B)}(x)$$

$$\text{Therefore} \quad \upsilon^+_{?(A\cap B)}(mx) \leq \upsilon^+_{?(A)\cap ?(B)}(x)$$

$$\text{Consider} \quad \mu^-_{?(A\cap B)}(mx) = \max\left(\frac{-1}{2}, \mu^-_{A\cap B}(mx)\right) \leq \max\left(\max\left(\frac{-1}{2}, \mu^-_A(x)\right), \max\left(\frac{-1}{2}, \mu^-_B(x)\right)\right)$$

$$= \max\left(\mu^-_{?A}(x), \mu^-_{?B}(x)\right) = \mu^-_{?(A)\cap ?(B)}(x)$$

$$\text{Therefore} \quad \mu^-_{?(A\cap B)}(mx) \leq \mu^+_{?(A)\cap ?(B)}(x)$$

Consider $\upsilon^{-}_{?(A\cap B)}(mx) = \min\left(\frac{-1}{2}, \upsilon^{-}_{A\cap B}(mx)\right) \geq \min\left(\min\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right), \min\left(\frac{-1}{2}, \upsilon^{-}_{B}(x)\right)\right)$

$$= \min\left(\upsilon^{-}_{?A}(x), \upsilon^{-}_{?B}(x)\right) = \upsilon^{-}_{?(A)\cap ?(B)}(x)$$

Therefore $\upsilon^{-}_{?(A\cap B)}(mx) \geq \upsilon^{-}_{?(A)\cap ?(B)}(x)$

Therefore $?(A\cap B) = ?(A)\cap ?(B)$ is an bipolar intuitionistic M fuzzy group of G.

Theorem 2.11. If A is an bipolar intuitionistic M fuzzy group of G then $?(\overline{\overline{A}}) = !(A)$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G$ *implies* $x, m \in A$

Consider $\mu^{+}_{?(\overline{\overline{A}})}(mx) = \upsilon^{+}_{?(\overline{A})}(mx) = \max\left(\frac{1}{2}, \upsilon^{+}_{\overline{A}}(mx)\right) \geq \max\left(\frac{1}{2}, \mu^{+}_{A}(x)\right) = \mu^{+}_{!(A)}(x)$

Therefore $\mu^{+}_{?(\overline{\overline{A}})}(mx) \geq \mu^{+}_{!(A)}(x)$

Consider $\upsilon^{+}_{?(\overline{\overline{A}})}(mx) = \mu^{+}_{?(\overline{A})}(mx) = \min\left(\frac{1}{2}, \mu^{+}_{\overline{A}}(mx)\right) \leq \min\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right) = \upsilon^{+}_{!(A)}(x)$

Therefore $\upsilon^{+}_{?(\overline{\overline{A}})}(mx) \leq \upsilon^{+}_{!(A)}(x)$

Consider $\mu^{-}_{?(\overline{\overline{A}})}(mx) = \upsilon^{-}_{?(\overline{A})}(mx) = \min\left(\frac{-1}{2}, \upsilon^{-}_{\overline{A}}(mx)\right) \leq \min\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right) = \mu^{-}_{!(A)}(x)$

Therefore $\mu^{-}_{?(\overline{\overline{A}})}(mx) \leq \mu^{-}_{!(A)}(x)$

Consider $\upsilon^{-}_{?(\overline{\overline{A}})}(mx) = \mu^{-}_{?(\overline{A})}(mx) = \max\left(\frac{-1}{2}, \mu^{-}_{\overline{A}}(mx)\right) \geq \max\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right) = \upsilon^{-}_{!(A)}(x)$

Therefore $\upsilon^{-}_{?(\overline{\overline{A}})}(mx) \geq \upsilon^{-}_{!(A)}(x)$

Therefore $?(\overline{\overline{A}}) = !(A)$ is an bipolar intuitionistic M fuzzy group of G.

Theorem 2.12. If A is an bipolar intuitionistic M fuzzy group of G then $!(?(A)) = ?(!(A))$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G$ *implies* $x, m \in A$

Consider $\mu^{+}_{!(?(A))}(mx) = \max\left(\frac{1}{2}, \mu^{+}_{?(A)}(mx)\right) = \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \mu^{+}_{A}(mx)\right)\right)$

$$\geq \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \mu^{+}_{A}(x)\right)\right) = \mu^{+}_{?(!(A))}(x)$$

Therefore $\mu^{+}_{!(?(A))}(mx) \geq \mu^{+}_{?(!(A))}(x)$

Consider $\upsilon^{+}_{!(?(A))}(mx) = \min\left(\frac{1}{2}, \upsilon^{+}_{?(A)}(mx)\right) = \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \upsilon^{+}_{A}(mx)\right)\right)$

$$\leq \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \upsilon^{+}_{A}(mx)\right)\right) = \upsilon^{+}_{?(!(A))}(x)$$

Therefore $\upsilon^{+}_{!(?(A))}(mx) \leq \upsilon^{+}_{?(!(A))}(x)$

Consider $\mu^{-}_{!(?(A))}(mx) = \min\left(\frac{-1}{2}, \mu^{-}_{?(A)}(mx)\right) = \min\left(\frac{-1}{2}, \max\left(\frac{-1}{2}, \mu^{-}_{A}(mx)\right)\right)$

$$\leq \max\left(\frac{-1}{2}, \min\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right)\right) = \mu^{-}_{?(!(A))}(x)$$

Therefore $\mu^{-}_{!(?(A))}(mx) \leq \mu^{-}_{?(!(A))}(x)$

Consider $\upsilon^{-}_{!(?(A))}(mx) = \max\left(\frac{-1}{2}, \upsilon^{-}_{?(A)}(mx)\right) = \max\left(\frac{-1}{2}, \min\left(\frac{-1}{2}, \upsilon^{-}_{A}(mx)\right)\right)$

$$\geq \min\left(\frac{-1}{2}, \max\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right)\right) = \upsilon^{-}_{?(!(A))}(x)$$

Therefore $\upsilon^{-}_{!(?(A))}(mx) \geq \upsilon^{-}_{?(!(A))}(x)$

Therefore $!(?(A)) = ?(!(A))$ is an bipolar intuitionistic M fuzzy group of G.

Theorem 2.13. If A is an bipolar intuitionistic M fuzzy group of G then $!(\square(A)) = \square(!(A))$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G$ *implies* $x, m \in A$

Consider $\mu^{+}_{!(\square A)}(mx) = max\left(\frac{1}{2}, \mu^{+}_{\square A}(mx)\right) \geq max\left(\frac{1}{2}, \mu^{+}_{A}(x)\right) = \mu^{+}_{\square(!(A))}(x)$

Therefore $\mu^{+}_{!(\square A)}(mx) \geq \mu^{+}_{\square(!(A))}(x)$

Consider $\upsilon^{+}_{!(\square(A))}(mx) = \min\left(\frac{1}{2}, \upsilon^{+}_{\square(A)}(mx)\right) \leq \min\left(\frac{1}{2}, 1 - \mu^{+}_{A}(x)\right) = \min\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right)$

$$= \upsilon^{+}_{!(A)}(x) = 1 - \mu^{+}_{\square(!(A))}(x) = \upsilon^{+}_{\square(!(A))}(x)$$

Therefore $\upsilon^{+}_{!(\square(A))}(mx) \leq \upsilon^{+}_{\square(!(A))}(x)$

Consider $\mu^{-}_{!(\square A)}(mx) = \min\left(\frac{-1}{2}, \mu^{-}_{\square A}(mx)\right) \leq \min\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right) = \mu^{-}_{\square(!(A))}(x)$

Therefore $\mu^{-}_{!(\square A)}(mx) \leq \mu^{-}_{\square(!(A))}(x)$

Consider $\upsilon^{-}_{!(\square(A))}(mx) = \max\left(\frac{-1}{2}, \upsilon^{-}_{\square(A)}(mx)\right) \geq \max\left(\frac{-1}{2}, -1 - \mu^{-}_{A}(x)\right) = \max\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right)$

$$= \upsilon^{-}_{!(A)}(x) = -1 - \mu^{-}_{\square(!(A))}(x) = \upsilon^{-}_{\square(!(A))}(x)$$

Therefore $\upsilon^{-}_{!(\square(A))}(mx) \geq \upsilon^{-}_{\square(!(A))}(x)$

Therefore $!(\square(A)) = \square(!(A))$ is an bipolar intuitionistic M fuzzy group of G.

Theorem 2.14. If A is an bipolar intuitionistic M fuzzy group of G then $?(\square(A)) = \square(?(A))$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G$ implies $x, m \in A$

Consider $\mu^{+}_{?(\square(A))}(mx) = \min\left(\frac{1}{2}, \mu^{+}_{\square A}(mx)\right) \geq \min\left(\frac{1}{2}, \mu^{+}_{A}(x)\right) = \mu^{+}_{?(A)}(x) = \mu^{+}_{\square(?(A))}(x)$

Therefore $\mu^{+}_{?(\square(A))}(mx) \geq \mu^{+}_{\square(?(A))}(x)$

$$\textit{Consider}\quad \upsilon^{+}_{?(\square(A))}(mx) = \max\left(\frac{1}{2}, \upsilon^{+}_{\square(A)}(mx)\right) \le \max\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right) = \upsilon^{+}_{?(A)}(x) = \upsilon^{+}_{\square(?(A))}(x)$$

$$\textit{Therefore}\quad \upsilon^{+}_{?(\square(A))}(mx) \le \upsilon^{+}_{\square(?(A))}(x)$$

$$\textit{Consider}\quad \mu^{-}_{?(\square(A))}(mx) = \max\left(\frac{-1}{2}, \mu^{-}_{\square A}(mx)\right) \le \max\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right) = \mu^{-}_{?(A)}(x) = \mu^{-}_{\square(?(A))}(x)$$

$$\textit{Therefore}\quad \mu^{-}_{?(\square(A))}(mx) \le \mu^{-}_{\square(?(A))}(x)$$

$$\textit{Consider}\quad \upsilon^{-}_{?(\square(A))}(mx) = \min\left(\frac{-1}{2}, \upsilon^{-}_{\square(A)}(mx)\right) \ge \min\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right) = \upsilon^{-}_{?(A)}(x) = \upsilon^{-}_{\square(?(A))}(x)$$

$$\textit{Therefore}\quad \upsilon^{-}_{?(\square(A))}(mx) \ge \upsilon^{+}_{\square(?(A))}(x)$$

Therefore $?(\square(A)) = \square(?(A))$ is an bipolar intuitionistic M fuzzy group of G.

Theorem 2.15. If A is an bipolar intuitionistic M fuzzy group of G then $!(\Diamond(A)) = \Diamond(!(A))$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G$ implies $x, m \in A$

$$\textit{Consider}\quad \mu^{+}_{!(\Diamond(A))}(mx) = \max\left(\frac{1}{2}, \mu^{+}_{\Diamond A}(mx)\right) \ge \max\left(\frac{1}{2}, 1-\upsilon^{+}_{A}(x)\right) = \mu^{+}_{!(A)}(x) = \mu^{+}_{\Diamond(!(A))}(x)$$

$$\textit{Therefore}\quad \mu^{+}_{!(\Diamond(A))}(mx) \ge \mu^{+}_{\Diamond(!(A))}(x)$$

$$\textit{Consider}\quad \upsilon^{+}_{!(\Diamond(A))}(mx) = \min\left(\frac{1}{2}, \upsilon^{+}_{\Diamond(A)}(mx)\right) \le \min\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right) = \upsilon^{+}_{!A}(x) = \upsilon^{+}_{\Diamond(!(A))}(x)$$

$$\textit{Therefore}\quad \upsilon^{+}_{!(\Diamond(A))}(mx) \le \upsilon^{+}_{\Diamond(!(A))}(x)$$

$$\textit{Consider}\quad \mu^{-}_{!(\Diamond(A))}(mx) = \min\left(\frac{-1}{2}, \mu^{-}_{\Diamond A}(mx)\right) \le \min\left(\frac{-1}{2}, -1-\upsilon^{-}_{A}(x)\right) = \mu^{-}_{!(A)}(x) = \mu^{-}_{\Diamond(!(A))}(x)$$

$$\textit{Therefore}\quad \mu^{-}_{!(\Diamond(A))}(mx) \le \mu^{-}_{\Diamond(!(A))}(x)$$

$$\textit{Consider}\quad \upsilon^{-}_{!(\Diamond(A))}(mx) = \max\left(\frac{-1}{2}, \upsilon^{-}_{\Diamond(A)}(mx)\right) \ge \max\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right) = \upsilon^{-}_{!A}(x) = \upsilon^{-}_{\Diamond(!(A))}(x)$$

$$\textit{Therefore}\quad \upsilon^{-}_{!(\Diamond(A))}(mx) \ge \upsilon^{-}_{\Diamond(!(A))}(x)$$

Therefore $!(\Diamond(A)) = \Diamond(!(A))$ is an bipolar intuitionistic M fuzzy group of G.

Theorem 2.16. If A is an bipolar intuitionistic M fuzzy group of G then $?(\Diamond(A)) = \Diamond(?(A))$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G$ implies $x, m \in A$

$$\textit{Consider}\quad \mu^{+}_{?(\Diamond(A))}(mx) = \min\left(\frac{1}{2}, \mu^{+}_{\Diamond A}(mx)\right) \ge \min\left(\frac{1}{2}, 1-\upsilon^{+}_{A}(x)\right) = \mu^{+}_{?(A)}(x)$$

$$= 1-\upsilon^{+}_{\Diamond(?(A))}(x) = \mu^{+}_{\Diamond(?(A))}(x)$$

$$\textit{Therefore}\quad \mu^{+}_{?(\Diamond(A))}(mx) \ge \mu^{+}_{\Diamond(?(A))}(x)$$

$$\textit{Consider}\quad \upsilon^{+}_{?(\Diamond(A))}(mx) = \max\left(\frac{1}{2}, \upsilon^{+}_{\Diamond(A)}(mx)\right) \le \max\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right) = \upsilon^{+}_{\Diamond(?(A))}(x)$$

$$\textit{Therefore}\quad \upsilon^{+}_{?(\Diamond(A))}(mx) \le \upsilon^{+}_{\Diamond(?(A))}(x)$$

Consider $\mu^{-}_{?(\Diamond(A))}(mx)=\max\left(\frac{-1}{2},\mu^{-}_{\Diamond A}(mx)\right)\leq\max\left(\frac{-1}{2},-1-\upsilon^{-}_{A}(x)\right)=\mu^{-}_{?(A)}(x)$

$$=-1-\upsilon^{-}_{\Diamond(?(A))}(x)=\mu^{-}_{\Diamond(?(A))}(x)$$

Therefore $\mu^{-}_{?(\Diamond(A))}(mx)\leq\mu^{-}_{\Diamond(?(A))}(x)$

Consider $\upsilon^{-}_{?(\Diamond(A))}(mx)=\min\left(\frac{-1}{2},\upsilon^{-}_{\Diamond(A)}(mx)\right)\geq\min\left(\frac{-1}{2},\upsilon^{-}_{A}(x)\right)=\upsilon^{-}_{\Diamond(?(A))}(x)$

Therefore $\upsilon^{-}_{?(\Diamond(A))}(mx)\geq\upsilon^{-}_{\Diamond(?(A))}(x)$

Therefore $?(\Diamond(A))=\Diamond(?(A))$ is an bipolar intuitionistic M fuzzy group of G.

3. Operators Over on Bipolar Intuitionistic anti M- fuzzy group of G. :

Theorem 3.1. If A is an bipolar intuitionistic anti M fuzzy group of G then !(A) is an bipolar intuitionistic anti M fuzzy group of G.

Proof. Consider $x,m\in G\ implies\ x,m\in A$

Consider $\mu^{+}_{!(A)}(mx)=\max\left(\frac{1}{2},\mu^{+}_{A}(mx)\right)\leq\max\left(\frac{1}{2},\mu^{+}_{A}(x)\right)=\mu^{+}_{!A}(x)$

Therefore $\mu^{+}_{!(A)}(mx)\leq\mu^{+}_{!A}(x)$ *for some* $m\in M$ *and* $x\in G$

Consider $\upsilon^{+}_{!(A)}(mx)=\min\left(\frac{1}{2},\upsilon^{+}_{A}(mx)\right)\geq\min\left(\frac{1}{2},\upsilon^{+}_{A}(x)\right)=\upsilon^{+}_{!A}(x)$

Therefore $\upsilon^{+}_{!(A)}(mx)\geq\upsilon^{+}_{!A}(x)$ *for some* $m\in M$ *and* $x\in G$

Consider $\mu^{-}_{!(A)}(mx)=\min\left(\frac{-1}{2},\mu^{-}_{A}(mx)\right)\geq\min\left(\frac{-1}{2},\mu^{-}_{A}(x)\right)=\mu^{-}_{!A}(x)$

Therefore $\mu^{-}_{!(A)}(mx)\geq\mu^{-}_{!A}(x)$ *for some* $m\in M$ *and* $x\in G$

Consider $\upsilon^{-}_{!(A)}(mx)=\max\left(\frac{-1}{2},\upsilon^{-}_{A}(mx)\right)\leq\max\left(\frac{-1}{2},\upsilon^{-}_{A}(x)\right)=\upsilon^{-}_{!A}(x)$

Therefore $\upsilon^{-}_{!(A)}(mx)\leq\upsilon^{-}_{!A}(x)$ *for some* $m\in M$ *and* $x\in G$

Therefore !(A) is an bipolar intuitionistic anti M fuzzy group of G.

Theorem 3.2. If A and B are bipolar intuitionistic anti M fuzzy group of G then $!(A\cap B)=!(A)\cap!(B)$ is an bipolar intuitionistic anti M fuzzy group of G.

Proof. Consider $x,m\in G\ implies\ x,m\in A\,and\ x,m\in B$

Consider $\mu^{+}_{!(A\cap B)}(mx)=\max\left(\frac{1}{2},\mu^{+}_{A\cap B}(mx)\right)\leq\min\left(\max\left(\frac{1}{2},\mu^{+}_{A}(x)\right),\max\left(\frac{1}{2},\mu^{+}_{B}(x)\right)\right)$

$$=\min\left(\mu^{+}_{!A}(x),\mu^{+}_{!B}(x)\right)=\mu^{+}_{!(A)\cap!(B)}(x)$$

Therefore $\mu^{+}_{!(A\cap B)}(mx)\leq\mu^{+}_{!(A)\cap!(B)}(x)$

Consider $\upsilon^{+}_{!(A\cap B)}(mx)=\min\left(\frac{1}{2},\upsilon^{+}_{A\cap B}(mx)\right)\geq\max\left(\min\left(\frac{1}{2},\upsilon^{+}_{A}(x)\right),\min\left(\frac{1}{2},\upsilon^{+}_{B}(x)\right)\right)$

$$=\max\left(\upsilon^{+}_{!A}(x),\upsilon^{+}_{!B}(x)\right)=\upsilon^{+}_{!(A)\cap!(B)}(x)$$

Therefore $\upsilon^{+}_{!(A\cap B)}(mx)\geq\upsilon^{+}_{!(A)\cap!(B)}(x)$

$$\textit{Consider} \quad \mu^{-}_{!(A\cap B)}(mx) = \min\left(\frac{-1}{2}, \mu^{-}_{A\cap B}(mx)\right) \geq \max\left(\min\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right), \min\left(\frac{-1}{2}, \mu^{-}_{B}(x)\right)\right)$$

$$= \max\left(\mu^{-}_{!A}(x), \mu^{-}_{!B}(x)\right) = \mu^{-}_{!(A)\cap !(B)}(x)$$

$$\textit{Therefore} \quad \mu^{-}_{!(A\cap B)}(mx) \geq \mu^{+}_{!(A)\cap !(B)}(x)$$

$$\textit{Consider} \quad \upsilon^{-}_{!(A\cap B)}(mx) = \max\left(\frac{-1}{2}, \upsilon^{-}_{A\cap B}(mx)\right) \leq \min\left(\max\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right), \max\left(\frac{-1}{2}, \upsilon^{-}_{B}(x)\right)\right)$$

$$= \min\left(\upsilon^{-}_{!A}(x), \upsilon^{-}_{!B}(x)\right) = \upsilon^{-}_{!(A)\cap !(B)}(x)$$

$$\textit{Therefore} \quad \upsilon^{-}_{!(A\cap B)}(mx) \leq \upsilon^{-}_{!(A)\cap !(B)}(x)$$

Therefore $!(A\cap B) = !(A)\cap !(B)$ is an bipolar intuitionistic anti M fuzzy group of G.

Theorem 3.3. If A is an bipolar intuitionistic anti M fuzzy group of G then $?(A)$ is an bipolar intuitionistic anti M fuzzy group of G.

Proof. Consider $x, m \in G \textit{ implies } x, m \in A$

$$\textit{Consider} \quad \mu^{+}_{?(A)}(mx) = \min\left(\frac{1}{2}, \mu^{+}_{A}(mx)\right) \leq \min\left(\frac{1}{2}, \mu^{+}_{A}(x)\right) = \mu^{+}_{?(A)}(x)$$

$$\textit{Therefore} \quad \mu^{+}_{?(A)}(mx) \leq \mu^{+}_{?(A)}(x) \quad \textit{for some } m \in M \textit{ and } x \in G$$

$$\textit{Consider} \quad \upsilon^{+}_{?(A)}(mx) = \max\left(\frac{1}{2}, \upsilon^{+}_{A}(mx)\right) \geq \max\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right) = \upsilon^{+}_{?(A)}(x)$$

$$\textit{Therefore} \quad \upsilon^{+}_{?(A)}(mx) \geq \upsilon^{+}_{?(A)}(x) \quad \textit{for some } m \in M \textit{ and } x \in G$$

$$\textit{Consider} \quad \mu^{-}_{?(A)}(mx) = \max\left(\frac{-1}{2}, \mu^{-}_{A}(mx)\right) \geq \max\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right) = \mu^{-}_{?(A)}(x)$$

$$\textit{Therefore} \quad \mu^{-}_{?(A)}(mx) \geq \mu^{-}_{?(A)}(x) \quad \textit{for some } m \in M \textit{ and } x \in G$$

$$\textit{Consider} \quad \upsilon^{-}_{?(A)}(mx) = \min\left(\frac{-1}{2}, \upsilon^{-}_{A}(mx)\right) \leq \min\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right) = \upsilon^{-}_{?(A)}(x)$$

$$\textit{Therefore} \quad \upsilon^{-}_{?(A)}(mx) \leq \upsilon^{-}_{?(A)}(x) \quad \textit{for some } m \in M \textit{ and } x \in G$$

Therefore $?(A)$ is an bipolar intuitionistic anti M fuzzy group of G.

Theorem 3.4. If A and B are bipolar intuitionistic anti M fuzzy group of G then $?(A\cap B) = ?(A)\cap ?(B)$ is an bipolar intuitionistic M fuzzy group of G.

Proof. Consider $x, m \in G \textit{ implies } x, m \in A \textit{ and } x, m \in B$

$$\textit{Consider} \quad \mu^{+}_{?(A\cap B)}(mx) = \min\left(\frac{1}{2}, \mu^{+}_{A\cap B}(mx)\right) \leq \min\left(\min\left(\frac{1}{2}, \mu^{+}_{A}(x)\right), \min\left(\frac{1}{2}, \mu^{+}_{B}(x)\right)\right)$$

$$= \min\left(\mu^{+}_{?A}(x), \mu^{+}_{?B}(x)\right) = \mu^{+}_{?(A)\cap ?(B)}(x)$$

$$\textit{Therefore} \quad \mu^{+}_{?(A\cap B)}(mx) \leq \mu^{+}_{?(A)\cap ?(B)}(x)$$

$$\textit{Consider} \quad \upsilon^{+}_{?(A\cap B)}(mx) = \max\left(\frac{1}{2}, \upsilon^{+}_{A\cap B}(mx)\right) \geq \max\left(\max\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right), \max\left(\frac{1}{2}, \upsilon^{+}_{B}(x)\right)\right)$$

$$= \max\left(\upsilon^{+}_{?A}(x), \upsilon^{+}_{?B}(x)\right) = \upsilon^{+}_{?(A)\cap ?(B)}(x)$$

$$\textit{Therefore} \quad \upsilon^{+}_{?(A\cap B)}(mx) \geq \upsilon^{+}_{?(A)\cap ?(B)}(x)$$

$$\textit{Consider} \quad \mu^{-}_{?(A\cap B)}(mx) = \max\left(\frac{-1}{2}, \mu^{-}_{A\cap B}(mx)\right) \geq \max\left(\max\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right), \max\left(\frac{-1}{2}, \mu^{-}_{B}(x)\right)\right)$$

$$= \max\left(\mu^{-}_{?A}(x), \mu^{-}_{?B}(x)\right) = \mu^{-}_{?(A)\cap ?(B)}(x)$$

$$\textit{Therefore} \quad \mu^{-}_{?(A\cap B)}(mx) \geq \mu^{+}_{?(A)\cap ?(B)}(x)$$

Consider $\upsilon^-_{?(A\cap B)}(mx) = \min\left(\frac{-1}{2}, \upsilon^-_{A\cap B}(mx)\right) \le \min\left(\min\left(\frac{-1}{2}, \upsilon^-_A(x)\right), \min\left(\frac{-1}{2}, \upsilon^-_B(x)\right)\right)$

$= \min\left(\upsilon^-_{?A}(x), \upsilon^-_{?B}(x)\right) = \upsilon^-_{?(A)\cap ?(B)}(x)$

Therefore $\upsilon^-_{?(A\cap B)}(mx) \le \upsilon^-_{?(A)\cap ?(B)}(x)$

Therefore $?(A\cap B) = ?(A)\cap ?(B)$ is an bipolar intuitionistic anti M fuzzy group of G.

Theorem 3.5. If A is an bipolar intuitionistic anti M fuzzy group of G then $?(\overline{\overline{A}}) = !(A)$ is an bipolar intuitionistic anti M fuzzy group of G.

Proof. Consider $x, m \in G$ *implies* $x, m \in A$

Consider $\mu^+_{?(\overline{\overline{A}})}(mx) = \upsilon^+_{?(\overline{A})}(mx) = \max\left(\frac{1}{2}, \mu^+_A(mx)\right) \le \max\left(\frac{1}{2}, \mu^+_A(x)\right) = \mu^+_{!(A)}(x)$

Therefore $\mu^+_{?(\overline{\overline{A}})}(mx) \le \mu^+_{!(A)}(x)$

Consider $\upsilon^+_{?(\overline{\overline{A}})}(mx) = \mu^+_{?(\overline{A})}(mx) = \min\left(\frac{1}{2}, \upsilon^+_A(mx)\right) \ge \min\left(\frac{1}{2}, \upsilon^+_A(x)\right) = \upsilon^+_{!(A)}(x)$

Therefore $\upsilon^+_{?(\overline{\overline{A}})}(mx) \ge \upsilon^+_{!(A)}(x)$

Consider $\mu^-_{?(\overline{\overline{A}})}(mx) = \upsilon^-_{?(\overline{A})}(mx) = \min\left(\frac{-1}{2}, \mu^-_A(mx)\right) \ge \min\left(\frac{-1}{2}, \mu^-_A(x)\right) = \mu^-_{!(A)}(x)$

Therefore $\mu^-_{?(\overline{\overline{A}})}(mx) \ge \mu^-_{!(A)}(x)$

Consider $\upsilon^-_{?(\overline{\overline{A}})}(mx) = \mu^-_{?(\overline{A})}(mx) = \max\left(\frac{-1}{2}, \upsilon^-_A(mx)\right) \le \max\left(\frac{-1}{2}, \upsilon^-_A(x)\right) = \upsilon^-_{!(A)}(x)$

Therefore $\upsilon^-_{?(\overline{\overline{A}})}(mx) \le \upsilon^-_{!(A)}(x)$

Therefore $?(\overline{\overline{A}}) = !(A)$ is an bipolar intuitionistic anti M fuzzy group of G.

Theorem 3.6. If A is an bipolar intuitionistic anti M fuzzy group of G then $!(?(A)) = ?(!(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

Proof. Consider $x, m \in G$ *implies* $x, m \in A$

Consider $\mu^+_{!(?(A))}(mx) = \max\left(\frac{1}{2}, \mu^+_{?(A)}(mx)\right) = \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \mu^+_A(mx)\right)\right)$

$\le \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \mu^+_A(x)\right)\right) = \mu^+_{?(!(A))}(x)$

Therefore $\mu^+_{!(?(A))}(mx) \le \mu^+_{?(!(A))}(x)$

Consider $\upsilon^+_{!(?(A))}(mx) = \min\left(\frac{1}{2}, \upsilon^+_{?(A)}(mx)\right) = \min\left(\frac{1}{2}, \max\left(\frac{1}{2}, \upsilon^+_A(mx)\right)\right)$

$\ge \max\left(\frac{1}{2}, \min\left(\frac{1}{2}, \upsilon^+_A(x)\right)\right) = \upsilon^+_{?(!(A))}(x)$

Therefore $\upsilon^+_{!(?(A))}(mx) \ge \upsilon^+_{?(!(A))}(x)$

Consider $\mu^-_{!(?(A))}(mx) = \min\left(\frac{-1}{2}, \mu^-_{?(A)}(mx)\right) = \min\left(\frac{-1}{2}, \max\left(\frac{-1}{2}, \mu^-_A(mx)\right)\right)$

$\ge \min\left(\frac{-1}{2}, \max\left(\frac{-1}{2}, \mu^-_A(x)\right)\right) = \mu^-_{?(!(A))}(x)$

Therefore $\mu^-_{!(?(A))}(mx) \ge \mu^-_{?(!(A))}(x)$

Consider $\upsilon^{-}_{!(?(A))}(mx) = \max\left(\frac{-1}{2}, \upsilon^{-}_{?(A)}(mx)\right) = \max\left(\frac{-1}{2}, \min\left(\frac{-1}{2}, \upsilon^{-}_{A}(mx)\right)\right)$

$$\leq \min\left(\frac{-1}{2}, \max\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right)\right) = \upsilon^{-}_{?(!(A))}(x)$$

Therefore $\upsilon^{-}_{!(?(A))}(mx) \leq \upsilon^{-}_{?(!(A))}(x)$

Therefore $!(?(A)) = ?(!(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

Theorem 3.7. If A is an bipolar intuitionistic anti M fuzzy group of G then $!(\square(A)) = \square(!(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

Proof. Consider $x, m \in G$ *implies* $x, m \in A$

Consider $\mu^{+}_{!(\square A)}(mx) = max\left(\frac{1}{2}, \mu^{+}_{\square A}(mx)\right) \leq max\left(\frac{1}{2}, \mu^{+}_{A}(x)\right) = \mu^{+}_{!(A)}(x) = \mu^{+}_{\square(!(A))}(x)$

Therefore $\mu^{+}_{!(\square A)}(mx) \leq \mu^{+}_{\square(!(A))}(x)$

Consider $\upsilon^{+}_{!(\square(A))}(mx) = \min\left(\frac{1}{2}, \upsilon^{+}_{\square(A)}(mx)\right) \geq \min\left(\frac{1}{2}, 1 - \mu^{+}_{A}(x)\right) = \upsilon^{+}_{!(A)}(x)$

$$= 1 - \mu^{+}_{\square(!(A))}(x) = \upsilon^{+}_{\square(!(A))}(x)$$

Therefore $\upsilon^{+}_{!(\square(A))}(mx) \geq \upsilon^{+}_{\square(!(A))}(x)$

Consider $\mu^{-}_{!(\square A)}(mx) = \min\left(\frac{-1}{2}, \mu^{-}_{\square A}(mx)\right) \geq \min\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right) = \mu^{-}_{\square(!(A))}(x)$

Therefore $\mu^{-}_{!(\square A)}(mx) \geq \mu^{-}_{\square(!(A))}(x)$

Consider $\upsilon^{-}_{!(\square(A))}(mx) = \max\left(\frac{-1}{2}, \upsilon^{-}_{\square(A)}(mx)\right) \leq \max\left(\frac{-1}{2}, -1 - \mu^{-}_{A}(x)\right) = \upsilon^{-}_{!(A)}(x)$

$$= -1 - \mu^{-}_{\square(!(A))}(x) = \upsilon^{-}_{\square(!(A))}(x)$$

Therefore $\upsilon^{-}_{!(\square(A))}(mx) \leq \upsilon^{-}_{\square(!(A))}(x)$

Therefore $!(\square(A)) = \square(!(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

Theorem 3.8. If A is an bipolar intuitionistic anti M fuzzy group of G then $?(\square(A)) = \square(?(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

Proof. Consider $x, m \in G$ implies $x, m \in A$

Consider $\mu^{+}_{?(\square A)}(mx) = \min\left(\frac{1}{2}, \mu^{+}_{\square A}(mx)\right) \leq \min\left(\frac{1}{2}, \mu^{+}_{A}(x)\right) = \mu^{+}_{?(A)}(x) = \mu^{+}_{\square(?(A))}(x)$

Therefore $\mu^{+}_{?(\square A)}(mx) \leq \mu^{+}_{\square(?(A))}(x)$

Consider $\upsilon^{+}_{?(\square A)}(mx) = \max\left(\frac{1}{2}, \upsilon^{+}_{\square(A)}(mx)\right) \geq \max\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right) = \upsilon^{+}_{?(A)}(x) = 1 - \mu^{+}_{\square(?(A))}(x)$

$$= \upsilon^{+}_{\square(?(A))}(x)$$

Therefore $\upsilon^{+}_{?(\square A)}(mx) \geq \upsilon^{+}_{\square(?(A))}(x)$

Consider $\mu^{-}_{?(\square A)}(mx) = \max\left(\frac{-1}{2}, \mu^{-}_{\square A}(mx)\right) \geq \max\left(\frac{-1}{2}, \mu^{-}_{A}(x)\right) = \mu^{-}_{?(A)}(x) = \mu^{-}_{\square(?(A))}(x)$

Therefore $\mu^{-}_{?(\square A)}(mx) \geq \mu^{-}_{\square(?(A))}(x)$

Consider $\upsilon^{-}_{?(\Box A)}(mx) = \min\left(\frac{-1}{2}, \upsilon^{-}_{\Box(A)}(mx)\right) \le \min\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right) = \upsilon^{-}_{?(A)}(x)$

$$= -1 - \mu^{-}_{\Box(?(A))}(x) = \upsilon^{-}_{\Box(?(A))}(x)$$

Therefore $\upsilon^{-}_{?(\Box A)}(mx) \le \upsilon^{+}_{\Box(?(A))}(x)$

Therefore $?(\Box(A)) = \Box(?(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

Theorem.3.9. If A is an bipolar intuitionistic anti M fuzzy group of G then $!(\Diamond(A)) = \Diamond(!(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

Proof. Consider $x, m \in G$ implies $x, m \in A$

Consider $\mu^{+}_{!(\Diamond A)}(mx) = \max\left(\frac{1}{2}, \mu^{+}_{\Diamond A}(mx)\right) \le \max\left(\frac{1}{2}, 1 - \upsilon^{+}_{A}(x)\right) = \mu^{+}_{!(A)}(x)$

$$= 1 - \upsilon^{+}_{\Diamond(!(A))}(x) = \mu^{+}_{\Diamond(!(A))}(x)$$

Therefore $\mu^{+}_{!(\Diamond A)}(mx) \le \mu^{+}_{\Diamond(!(A))}(x)$

Consider $\upsilon^{+}_{!(\Diamond A)}(mx) = \min\left(\frac{1}{2}, \upsilon^{+}_{\Diamond(A)}(mx)\right) \ge \min\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right) = \upsilon^{+}_{!A}(x) = \upsilon^{+}_{\Diamond(!(A))}(x)$

Therefore $\upsilon^{+}_{!(\Diamond A)}(mx) \ge \upsilon^{+}_{\Diamond(!(A))}(x)$

Consider $\mu^{-}_{!(\Diamond A)}(mx) = \min\left(\frac{-1}{2}, \mu^{-}_{\Diamond A}(mx)\right) \ge \min\left(\frac{-1}{2}, -1 - \upsilon^{-}_{A}(x)\right) = \mu^{-}_{!(A)}(x)$

$$= -1 - \upsilon^{-}_{\Diamond(!(A))}(x) = \mu^{-}_{\Diamond(!(A))}(x)$$

Therefore $\mu^{-}_{!(\Diamond A)}(mx) \ge \mu^{-}_{\Diamond(!(A))}(x)$

Consider $\upsilon^{-}_{!(\Diamond A)}(mx) = \max\left(\frac{-1}{2}, \upsilon^{-}_{\Diamond(A)}(mx)\right) \le \max\left(\frac{-1}{2}, \upsilon^{-}_{A}(x)\right) = \upsilon^{-}_{!A}(x) = \upsilon^{-}_{\Diamond(!(A))}(x)$

Therefore $\upsilon^{-}_{!(\Diamond A)}(mx) \le \upsilon^{-}_{\Diamond(!(A))}(x)$

Therefore $!(\Diamond(A)) = \Diamond(!(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

Theorem 3.10. If A is an bipolar intuitionistic anti M fuzzy group of G then $?(\Diamond(A)) = \Diamond(?(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

Proof. Consider $x, m \in G$ implies $x, m \in A$

Consider $\mu^{+}_{?(\Diamond A)}(mx) = \min\left(\frac{1}{2}, \mu^{+}_{\Diamond A}(mx)\right) \le \min\left(\frac{1}{2}, 1 - \upsilon^{+}_{A}(x)\right) = \mu^{+}_{?(A)}(x)$

$$= 1 - \upsilon^{+}_{\Diamond(?(A))}(x) = \mu^{+}_{\Diamond(?(A))}(x)$$

Therefore $\mu^{+}_{?(\Diamond A)}(mx) \le \mu^{+}_{\Diamond(?(A))}(x)$

Consider $\upsilon^{+}_{?(\Diamond A)}(mx) = \max\left(\frac{1}{2}, \upsilon^{+}_{\Diamond(A)}(mx)\right) \ge \max\left(\frac{1}{2}, \upsilon^{+}_{A}(x)\right) = \upsilon^{+}_{\Diamond(?(A))}(x)$

Therefore $\upsilon^{+}_{?(\Diamond A)}(mx) \ge \upsilon^{+}_{\Diamond(?(A))}(x)$

Consider $\mu^{-}_{?(\Diamond A)}(mx) = \max\left(\frac{-1}{2}, \mu^{-}_{\Diamond A}(mx)\right) \ge \max\left(\frac{-1}{2}, -1 - \upsilon^{-}_{A}(x)\right) = \mu^{-}_{?(A)}(x)$

$$= -1 - \upsilon^{-}_{\Diamond(?(A))}(x) = \mu^{-}_{\Diamond(?(A))}(x)$$

Therefore $\mu^{-}_{?(\Diamond A)}(mx) \ge \mu^{-}_{\Diamond(?(A))}(x)$

Consider $\upsilon^-_{?(\Diamond A)}(mx) = \min\left(\frac{-1}{2}, \upsilon^-_{\Diamond(A)}(mx)\right) \le \min\left(\frac{-1}{2}, \upsilon^-_A(x)\right) = \upsilon^-_{\Diamond(?(A))}(x)$

Therefore $\upsilon^-_{?(\Diamond A)}(mx) \le \upsilon^-_{\Diamond(?(A))}(x)$

Therefore $?(\Diamond(A)) = \Diamond(?(A))$ is an bipolar intuitionistic anti M fuzzy group of G.

4. Conclusion

The concept of bipolar intuitionistic M fuzzy group and anti M fuzzy group are defined and new algebraic structure of a bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group are created and some related properties and some operations of level operators are investigated. The purpose of the study is to implement fuzzy set theory and group theory of bipolar intuitionistic M fuzzy group and anti M fuzzy group. We hope that our results can also be extended to other algebraic system.

5. References

1. Atanassov. K.T., Intuitionistic fuzzy sets, Fuzzy sets and systems, pp. 207-296 (1986).
2. Atanassov. K.T., New operations defined over intuitionistic fuzzy sets, Fuzzy sets and systems, vol. *61*. No. *2*, pp. 137-142.
3. Atanassov. K.T., Inutitionistic fuzzy sets, Fuzzy sets and systems, vol. *20*, No.*1*, pp. 87-96 (1986).
4. Atanassov. K.T., Operators over interval valued intuitionistic fuzzy sets, Fuzzy sets and systems, vol. *64*, No.*2*, pp. 159-174 (1994).
5. Atanassov. K.T., Intuitionistic fuzzy sets; Theory and Applications, springer-verlag (1999).
6. Atanassov. K.T., Level operators on intuitionistic fuzzy sets, BUSEFAL, vol. *54*, pp. 4-8, (1993).
7. Atanassov. K.T., Four new operators on intuitionistic fuzzy sets, preprint IM-MFAIS pp. 4-89, sofia (1989).
8. Lee. K.M., Bipolar valued fuzzy sets and their operations, Proc. int. con. onintelligent and technologies, Bangkok, Thailand, cz.
9. Muthuraj. R., Sridharan. M., Bipolar anti fuzzy HX groups and its lower level sub HX subgroups, Journal ofphysical sciene, vol. *16*, pp.157-1699, (2012).
10. Palaniappan. N., Muthuraj. R., Anti fuzzy group and lower level subgroups, Antarctica J. Math., vol. *1(1)*, pp.71-76, (2004).
11. Palanivelrajan. M., Nandakumar. S., Some properties of intuitionistic fuzzy primary and fuzzy semi primary ideals, Notes on intuitionistic fuzzy sets, vol. *18*, No. 3, pp. 68-74, (2012).
12. Palanivelrajan. M., Nandakumar. S., Some operations of intuitionistic fuzzy primary and semi primary ideal, Asian journal of Algebra, vol. *5*, No. *2*, pp. 44-49, (2012).
13. Palanivelrajan. M., Gunasekaran. K., and Nandakumar. S., Level operators over intuitionistic fuzzy primary ideal and semi primary ideal, Advances in Fuzzy sets and systems, vol. *15*, No. *2*, pp 95-111, (2013).
14. Rosenfeld. A., Fuzzy groups, J. math. anal. appln., vol. *35*, pp. 512-517, (1971).
15. Zadeh. L.A., Fuzzy sets, Information and control,vol. *8*, pp. 338-365, (1965).
16. Zhang. W.R., Bipolar Fuzzy sets, Proc. of fuzz-IEEE, pp. 835-840, (1998).