Irresoluteness via binary supra topological spaces

M. LELLIS THIVAGAR[1] and J. KAVITHA[2]

[1,2]School of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu (India)
Corresponding Author Email:- mlthivagar@yahoo.co.in,   kavi.asm08@gmail.com

Abstract

In this paper we introduce the new class of functions called binary supra α-irresolute functions and deals with the concrete examples. The purpose of the present note is to introduce a strong form of binary supra continuity called binary supra strongly α-irresolute which is stronger than binary supra α-irresolute. Basic and some comparitive properties of these fuctions are studied in this paper.

Key words: Supra topology, Binary topology, Binary continuous, irresolute mappings, completely α-irresolute.

1. Introduction

In 1983 A.S. Mashhour *et al.*[7] inroduced the supra topological space and studied supra continuous maps. The class of strongly continuous functions were defined by N. Levine[5] in 1960 and that of α-continuous functions were defined by A.S. Mashhour *et al.*[7]. The class of α- irresolute functions were defined and studied by S.N. Maheshwari *et.al.,* in[6]. The class of α-irresolute function is weaker than the class of strongly continuous function but stronger than that of α-continuous function. Govindappa Navalagi[2] studied the class of completely α irresolute function. A function $f : X \rightarrow Y$ is said to be strongly continuous (strongly α-irrresolute, completely α-irresolute) if the inverse image each subset of Y is clopen in X (if the inverse image of each α-open set in Y is open in X, if the inverse image of each α-open set of Y is regular open in X). A binary topology from X to Y is a binary structure $M \subseteq P(X) \times P(Y)$ that satisfies the following axioms. (i) The empty set and whole space are binary open. (ii) the intersection of two (finite) binary open sets is an binary open set. (iii) the union of arbitrary collection of binary open sets is an binary open set is given in[9]. In this paper we introduce binary supra α-irresolute functions and also we study about completely α- irresolute and strongly α-irresolute functions in binary supra topological space.

2 Preliminaries:
Notation 2.1[9]:

i. $(A, B)^{1^*} = \cap \{ A_\alpha : (A_\alpha, B_\alpha) $ is binary closed and (A,B) $\subseteq (A_\alpha, B_\alpha)\}.$·

ii. $(A, B)^{2^*} = \cap \{ B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A,B) \subseteq (A_\alpha, B_\alpha)\}$.

Definition 2.2[9]: The ordered pair $((A, B)^{1^*}, (A, B)^{2^*})$ is called the binary closure of (A,B), denoted by B-cl(A,B) in the binary space (X,Y,M) where $(A,B) \subseteq (X,Y)$.

Notation 2.3[9]:

i. $(A, B)^{1^o} = \cup \{ A_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.

ii. $(A, B)^{2^o} = \cup \{ B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.

Definition 2.4[9]: Let (X,Y,M) be a binary topological space and $(A,B) \subseteq (X,Y)$. The ordered pair $((A, B)^{1^o}, (A, B)^{2^o})$ is called the binary interior of (A,B) denoted by B-int(A,B).

Definition 2.5[9]: Let $f : Z \rightarrow X \times Y$ be a function. Let $A \subseteq X$ and $B \subseteq Y$. We define $f^{-1}(A, B) = \{ z \in Z : f(z) = (x, y) \in (A,B)\}$ is binary continuous

Definition 2.6[4]: Let (X,Y,M) be a binary topological space, let (Z,τ) be a topological space and μ be an supra topology associated with τ. Let $f : Z \rightarrow X \times Y$ be a function. Then f is called binary supra continuous if $f^{-1}(A, B)$ is supra open in Z for every binary open set (A,B) in $X \times Y$.

Definition 2.7[4]: Let (X,Y,M) be a binary topological space, (Z,τ) be a topological space and μ be an supra topology associated with τ. Let $f : Z \rightarrow X \times Y$ be a function. Let $f : Z \rightarrow X \times Y$ be a function then:

i. f is called binary supra α-continuous if $f^{-1}(A, B)$ is supra α-open in Z for every binary open set (A, B) in $X \times Y$.

ii. f is called binary supra semicontinuous if $f^{-1}(A, B)$ is supra semiopen in Z for every binary open set (A,B) in $X \times Y$.

iii. f is called binary supra precontinuous if $f^{-1}(A, B)$ is supra preopen in Z for every binary open set (A,B) in $X \times Y$.

Theorem 2.8[4]: Let (X,Y,M) is a binary topological spaces, (Z,τ) is a topological space and μ be an associated supra topology with τ. Let $f : Z \rightarrow X \times Y$ be a function such that $Z \backslash f^{-1}(A, B) = f^{-1}(X \backslash A, Y \backslash B)$ for all $A \subseteq X$ and $B \subseteq Y$. Then f is binary supra continuous iff $f^{-1}(A, B)$ is supra closed in Z for all binary closed sets (A,B) in (X,Y, B_μ).

3 Binary supra α- irresolute mappings :

This section deals with the basic concepts of Binary supra α-irresolute function.

Definition 3.1 : Let (X,Y,M) be a binary topological space, (Z, τ) be a topological space and μ be an supra topology associated with τ. A function $f : Z \rightarrow X \times Y$ is binary supra α-irresolute if the inverse image of every α-set in $X \times Y$ is an α_μ set in Z.

Example 3.2: Let X={a,b}, Y={1,2} and Z = {a,b,c,d}, M ={(X, Y), (∅, ∅), ({a}, {1})}, M$_\alpha$ = {(X, Y), (∅, ∅), (X, {1}), ({a}, Y), ({b}, {2})} τ = {Z, ∅, {a}, {b, c}, {a, b, c}}, μ={Z, ∅, {a}, {b, c}, {a, b}, {a, b, c}}, α_μ= {Z, ∅, {a}, {a, b}, {b, c}, {a, b, c}, {a, b, d}}. Define $f : Z \rightarrow X \times Y$ where $f(a) = (a,1), f(b) = (b,1), f(c) = (a,2), f(d) = (b,2)$. Here f is binary supra α-continuous but not binary supra α-irresolute.

Theorem 3.3: Let (X,Y,M) be a binary topological space, (Z,τ) be a topological space and μ be an supra topology associated with τ. Let $f : Z \rightarrow X \times Y$ be a function such that $Z \backslash f^{-1}(A, B) = f^{-1}(X \backslash A, Y \backslash B)$ for all $A \subseteq X$ and $B \subseteq Y$. Then f is binary supra α-irresolute if and only if $f^{-1}(A, B)$ is supra α-closed in Z for all binary α-closed sets (A, B) in (X,Y,M).

Proof : Assume thet f is binary supra irresolute. Let (A,B) $\in X \times Y$ be a binary α-closed set. Therefore (X\A,Y\B) is binary α-open set. That is (X\A,Y\B) $\in$ M. Since f is binary supra irresolute, we have $f^{-1}(X \backslash A, Y \backslash B)$ is supra α-open in Z. Therefore $Z \backslash f^{-1}(A,B)$ is supra α-open in Z. Hence, $f^{-1}(A,B)$ is supra α-closed in Z. Conversely, assume that if $f^{-1}(A, B)$ is supra α-closed in Z for all binary α-closed set (A,B) in (X,Y,M). Let (A,B) $\in X \times Y$ be a binary α-open set. To prove $f^{-1}(A, B)$ is supra α-open in Z. Since (A,B) $\in$ M, we have (X\A,Y\B) is binary α-closed set in $X \times Y$. Therefore, by our assumption $f^{-1}(X \backslash A, Y \backslash B)$ is supra α-closed in Z. Thus, $Z f^{-1}(A,B)$ is supra α-closed in Z. Hence $f^{-1}(A, B)$ is supra α-open in Z. This proves that f is binary supra irresolute.

Definition 3.4: Let (X,Y,M) be a binary topological space, (Z,τ) be a topological space and μ be an supra

topology associated with τ. Let $f : Z \to X \times Y$ be a function. The function $f : Z \to X \times Y$ is called,

i.   binary supra irresolute if the inverse image of each supra semiopen set in $X \times Y$ is supra semiopen in Z.

ii. binary supra pre-irresolute if the inverse image of each supra preopen set in $X \times Y$ is supra preopen in Z.

Theorem 3.5: If $f : Z \to X \times Y$ is binary supra semiirresolute and binary supra preirresolute then f is binary supra α-irresolute.

Proof : If (U,V) is an binary α-set in $X \times Y$. Since f is irresolute and $(U,V) \in BSO(X,Y)$ we have $f^{-1}(U,V) \in SO_\mu(Z)$. Similarly, f is binary supra preirresolute and $(U,V) \in BPO(X,Y)$ implies $f^{-1}(U,V) \in PO_\mu(Z)$. Hence $f^{-1}(U,V) \in \alpha_\mu$, so that f is $B_s\alpha$-irresolute. Converse of the theorem is false by the following example.

Example 3.6: Let X={a,b,c}, Z={p,q,r} and M={$(X,X),(\emptyset,\emptyset),(\{b\},\{a\}),(\{b,c\},\{a\}),(\{b,c\},\{a,b\})$},

$\tau = \{Z, \emptyset, \{p, r\}\}$. μ ={$Z, \emptyset, \{p, q\}, \{p, r\}$}. Define a function $f : Z \to X \times Y$ as f(p) = (b,a), f(q)=(b,a), f(r) = (a,c). Here $f^{-1}(\{a, c\}, \{a, c\})$ = {r} is not supra preopen in Z.

4 Binary supra strongly α-irresolute mappings :

This section gives the charaterisations of binary supra strongly α- irresolute function and comparisons.

Definition 4.1 : Let (X,Y,M) be a binary topological space, let (Z,τ) be a topological space and μ be an supra topology associated with τ. A mapping $f : Z \to X \times Y$ is said to be binary supra strongly α-irresolute if the inverse image of each binary α-open set in $X \times Y$ is supra open in Z.

Example 4.2 : Let X = {a,b}, Y = {1,2}, Z ={x,y,z}, M = {$(X,Y),(\emptyset,\emptyset),(\{a\},\{1\})$}, M_α = {(X, Y), $(\emptyset,\emptyset)$, (X, 1}), $(\{a\}, Y), (\{a\}, \{1\})$}, $\tau = \{Z, \emptyset, \{x\}\}$, $\mu = \{Z, \emptyset, \{x\}, \{x, y\}, \{y, z\}\}$. Define a function $f : Z \to X \times Y$ by f(x)=(a,1), f(y) = (b,2) = f(z). Here f is binary supra strongly α-irresolute.

Remark 4.3 : The concept of binary supra strongly α-irresolute and binary supra irresolute are independent of each other. The following example shows that there exists a binary supra strongly α-irresolute mapping that is not irresolute.

Example 4.4 : Let X={a,b,c} and Z={a,b,c,d}, M={$(X,X),(\emptyset,\emptyset),(X,\{a\}),(\{c\},\{a\}),(X,\{a,c\}),(\{b,c\},\{a,c\}),$ $(\{b\},\emptyset),(\{b,c\},\{a\})$}, M_α = {$(X,X),(\emptyset,\emptyset),(X,\{a\}),(X,\{a,b\}),(X,\{a,c\})$, $(\{b\},\emptyset),(\{c\},\{a\}),(\{b,c\},X)$, $(\{b,c\},\{a\}),(\{b,c\},\{a,b\}),(\{b,c\},\{a,c\})$}, BSO(X,X) = {$(X,X),(\emptyset,\emptyset),(X,\{a\})$, $(X,\{a,b\}),(X,\{a,c\})$, $(\{b\},\emptyset),(\{b\},\{b\}),(\{b\},\{c\}),(\{c\},X),(\{c\},\{a\}),(\{c\},\{a,b\})$, $(\{c\},\{a,c\}),(\{a,b\},\emptyset),(\{a,b\},\{b\}),(\{a,b\},\{c\})$, $(\{a,b\},\{b,c\}),(\{a,c\},X),(\{a,c\},\{a\}),(\{a,c\},\{a,b\}),(\{a,c\},\{a,c\}),(\{b,c\},X),(\{b,c\},\{a\}),(\{b,c\},\{a,b\})$, $(\{b,c\},\{a,c\})$}, τ={Z, $\emptyset$,{a, b}, {c, d}, μ = {Z, $\emptyset$,{a, b}, {c, d}, {a, c, d}}, μ = μ semiopen. Define a function $f : Z \to X \times Y$ by f(a)=(b, b), f(b)=(a, b), f(c)=(c, a), f(d)=(c, a). Here f is binary supra strongly α-irresolute but it is not irresolute function since $f^{-1}(\{b\}, \{b\})$ = {a} is not supra semiopen.

Theorem 4.5 : Let (Z, τ) be a supra topological space and (X,Y,M) be a binary topological space. Let $f :(Z,\mu) \to$ (X,Y,M) be a mapping such that $Z \backslash f^{-1}(A, B) = f^{-1}(X \backslash A, Y \backslash B)$ for all $A \subseteq X$ and $B \subseteq Y$, then the following are equivalent:

i. f is binary supra strongly α-irresolute.

ii. $f : (Z,\mu) \to (X,Y, B\alpha))$ is binary supra continuous.

iii. the inverse image of each binary α-open set is supra open.

iv. the inverse image of each binary α-closed set is supra closed.

v. $cl_\mu(f^{-1}(B\alpha(A, B))) \subseteq f^{-1}(B\alpha - cl(A, B))$, for each $(A,B) \subseteq (X,Y)$.

Proof : (i) $\Rightarrow$ (ii): We know that inverse image of binary α-open set in $X \times Y$ is open in Z. Since we are considering binary α-open set and hence inverse image of α-open in $X \times Y$ is supra open in Z.

(ii)$\Rightarrow$(iii): is obvious.

f Assume f is binary supra strongly continuous. Let (A,B) ∈ X × Y be a binary α-closed set. Therefore, (X\A,Y\B) is binary α-open set. That is, (X\A,Y\B) ∈. *Bα*. Since *f* is binary supra strongly continuous, we have f^{-1}(X\A, Y\B) is supra open in Z. Therefore Z\ f^{-1}(A,B) is supra closed in Z.

(iii)⇒(i) : is trivial.

(iv)⇒(v): Since (A,B) is binary α-closed in X × Y, then it follows that f^{-1}(Bα–cl(A, B)) is supra closed in Z. Therefore, $f^{-1}(Bα–cl(A, B)) =μcl(f^{-1}(Bα–cl(A, B))) \supseteq cl_\mu(f^{-1}(Bα(A, B)))$.

Theorem 4.6: If a mapping $f : Z → X × Y$ is binary supra strongly α-irresolute, then $f^{-1}(U,V)$ is supra closed for any binary nowhere dense subset (U,V) of X × Y.

Proof : If (U,V) is binary nowhere dense subset of X × Y, then (X,Y)-(U,V) is an binary α-open set of X × Y and hence $f^{-1}((X,Y) − (U,V)) ∈$ M. It follows that $f^{-1}(U,V)$ is supra closed in Z.

Theorem 4.7: If $f : Z⇛ X × Y$ be a mapping, then the folowing are equivalent:
i. *f* is binary supra strongly α-irresolute.
ii. *f* is binary supra continuous and the inverse image of each nowhere dense set is supra closed.

Proof : (i) ⇒ (ii) It follows from the previous lemma.
(ii) ⇒ (i) Let (A,B) be an binary α-open in X × Y, then (A,B) may be written as a difference between an open set (C,D) and a binary nowhere dense set (E,F) ⊆ (C,D). Since $f^{-1}(A,B) = f^{-1}(C,D)- f^{-1}(E,F) = f^{-1}(C,D) ∪ (X-f^{-1}(E,F)) ∈$ M. Thus the proof is complete.

Theorem 4.8: Let $f : Z → X × Y$ be a mapping, then the following are equivalent:
i. *f* is binary supra strongly α-irresolute.
ii. For each binary points in (x,y) ∈ (X,Y) and each binary open set (U,V) ⊆ X × Y such that (x,y) ∈ B-int(B-cl(U,V)), the inverse image of (U,V) ∪ ({x},{y}) is a open subset of Z.

Proof : (i) ⇒ (ii) We have (U,V) ⊆ (U,V) ∪ ({x},{y}) ⊆ B-int(B-cl(U,V)) and then (U,V) ∪ ({x},{y}) is an $B_α$-open in X × Y and hence $f^{-1}((U,V) ∪ ({x},{y}) ∈ μ$.
(ii) ⇒ (i) Let (A,B) be an $B_α$-open subset of X × Y. There exists an binary open set (U,V) ⊆ (X,Y) such that (U,V) ⊆ (A,B) ⊆ B-int(B-cl(U,V)) and then for each (x,y) ∈ (A,B), $f^{-1}(U,V) ∪ ({x},{y}) ∈ μ$. Since f^{-1} (A, B) = $∪_{({x},{y})∈(A,B)} f^{-1}((U,V) ∪ ({x},{y}) ∈ μ$. Thus *f* is binary supra α-irresolute.

Remark 4.9: (X,Y,M) is called an binary α-space if M = $M_α$.

Theorem 4.10 : Let (X,Y) be a binary regular α-space. The following statements are equivalent:
i. $f : Z → X × Y$ is binary supra strongly α- irresolute.
ii. $f : Z → X × Y$ is binary supra α-irresolute.
iii. $f : Z → X × Y$ is binary supra α-continuous.

Proof : (i) ⇒ (ii): If *f* is binary supra strongly α-irresolute that is inverse image of each α-open in X × Y is supra open in Z. Since X × Y is regular α-space it is obvious that *f* is binary supra α- irresolute.
(ii) ⇒ (iii): Since every binary open in X × Y is binary α-open in X × Y and every binary regular open is binary open and hence *f* is binary supra α-continuous.
(iii) ⇒ (i): Since (X,Y) is binary regular space, *f* is binary supra continuous and then $f : Z → X × Y$ is binary supra continuous. Since it is reguar α-space. Therefore $f : Z → X × Y$ is binary supra strongly α-irresolute.

Definition 4.11: A binary topological space (X,Y,M) is binary connected if the only nonempty subset of (X,Y,M) which is both binary open and binary closed is (X,Y) itself.

Definition 4.12: A binary topological space (X,Y,M) is disconnected iff there exists nonempty proper subset of (X,Y,M) is both binary open and binary closed.

Theorem 4.13: A binary supra strongly α-irresolute image of a supra connected space is binary connected.

Proof : Let (Z,μ) and (X,Y,M) be an supra topological space and binary topological space and $f : Z → X × Y$ is

binary supra strongly α- irresolute function. In case (Z,μ) be supra connected then we have to prove that (X,Y,M) is also binary connected. Let (X,Y,M) be binary disconnected, then there exists a nonempty proper subset (A,B) of X × Y which is both binary open and binary closed. But f being binary supra strongly α-irresolute therefore f^{-1} (A,B) is both supra open and supra closed. Also f is onto mapping and (A,B) is proper nonempty subset of X × Y. Therefore f^{-1} (A,B) is also nonempty proper subset of Z which is shown above as both supra open and supra closed. Hence (Z,μ) is supra disconnected. Thus (X,Y,M) is binary disconnected ⇒ (Z,μ) is supra disconnected or (Z,μ) is supra connected ⇒ (X,Y,M) is binary connected.

Definition 4.14: Let (X,Y,M) be a binary topological space, let (Z,τ) be a topological space and μ be an supra topology associated with τ. A function f :Z → X × Y is called binary supra completely α-irresolute if the inverse image of each binary α-open set in X × Y is regular supra open in Z.

Remark 4.15: Clearly, every binary supra strongly continuous function is binary supra completely α-irresolute function and every binary supra completely α-irresolute function is binary supra completely continuous. Also, every binary supra completely α-irresolute function is binary supra α- irresolute function.

Theorem 4.16: Every binary supra completely α-irresolute function is binary supra strongly α-irresolute function.

*Proof :*If (U,V) is an binary α-open in X × Y. Since f is completely α-irresolute and (U,V) ∈ Bα(X,Y) we have $f^{-1}(U,V)$ is supra regular open in Z. Since every supra regular open is supra open and this implies inverse image of binary α-open is supra open in Z. Therefore f is binary supra strongly α-irresolute.

Remark 4.17: Converse of the Theorem 4.16 is not true by the following example.

Example 4.18: Let X = {a,b}, Y = {1,2} and Z={x,y,z}, M={(X,Y),(∅,∅), ({a},{1})} M_α={(X,Y),(∅,∅), (X,{1}),({a},Y),
({a},{1})}, μ = {Z,∅,{x},{x,y}{y,z}}. RO_μ = {Z,∅,{x},{y,z}}. Define a function f :Z → X × Y by f (x) = (a,1) = f (y), f (z) = (b,2). Here f is binary supra strongly α-irresolute but not completely α-irresolute since f^{-1}(X,{1}) is {x,y} is not supra regular open.

Definition 4.19: Let (X,Y,M) be a binary topological space, (Z,τ) be a topological space and μ be an supra topology associated with τ. Let f :Z → X × Y be a function. A function f :Z → X × Y is called binary supra strongly continuous if the inverse image of each subset of X × Y is supra clopen in Z.

Remark 4.20 : For a function f :Z → X × Y, the following implication are known
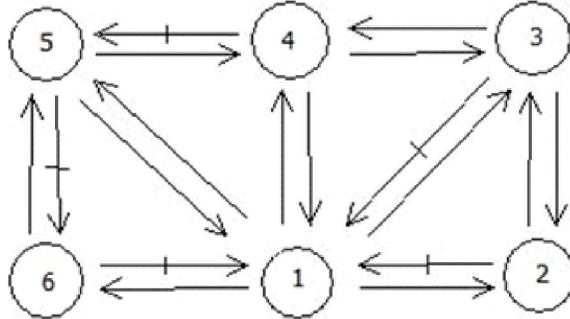
1. Binary supra continuous, 2. Binary supra α-continuous, 3. Binary supra α-irresolute, 4. Binary supra strongly α-irresolute, 5. Binary supra completely α-irresolute, 6. Binary supra strongly continuous.

Example 4.21: Let X={a,b,c}, Z={a,b,c,d}, M = {(X,Y),(∅,∅)(X,{a}),({c},{a}),(X,{a,c}), ({b,c},{a,c}), ({b},∅), ({b,c}, {a})}, τ = {Z,∅,{a},{b,c},{a,b,c}}, μ = {Z,∅,{a},{b,c},{a,c},{a,b,c}}. Define a function f :Z → X × Y by f (a)=(a,b), f (b)=(b,a)=f (c), f (d)=(c,b). Here f is not binary supra strongly continuous since f^{-1}(X,{a}) = {b,c} is supra open but not supra closed but f is binary supra continuous.

Example 4.22: Let X = {a,b,c}, Z = {p,q,r,s}, M={(X,Y),(∅,∅),({b},{a}), ({b,c},{a}),({b,c},{a,b})},

M$_\alpha$ = {(X,X), (∅,∅),(X,{a}), (X,{a,b},(X,{a,c}),({b},X),({b},{a}),({b},{a,b}),({b},{a,c}), ({a,b},X),({a,b},{a}),({a,b},{a,b}),({a,b},{a,c}),({b,c},X),({b,c},{a}), ({b,c},{a,b}), ({b,c},{a,c})}, τ ={Z,∅,{p},{p,q,r}}, μ = {Z,∅,{p},{q,r}, {p,r}, {p,q,r}}, μ$_\alpha$= {Z,∅,{p},{q,r}, {p,r}, {p,q,r},{p,r,s}}. Let f :Z → X × Y be a function defined by f (p)=(b,a)= f (r) = f (s), f (q) = (a,c). Since f^{-1}(X,{a}) = {p,r,s} is not open then the function is not binary supra strongly α-irresolute function but f is binary supra α-continuous.

Conclusion

General topology is important in many fields of applied sciences as well as branch of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, digital topology, informtion systems, non-commutative geometry and its application to quantam physics etc. Also this can be applied in binary supra topology. Binary supra α-irresolute functions has been applied in medical fields. The concept of the paper is to give new results concerning binary -irresolute functions. Additional strong form of binary supra continuiuty is discussed. Further our work can be extended to generalisation of continuity.

References

1. El-Deeb. S.N., Hasanein. I.A., Mashhour. A.S., Noiri. T., On p-regular spaces, *Bull. Math.de la Soc. Sci R.S.R Tome* (1983).
2. Govindappa Navalagi, On Completely α-irresolute functions., *International Journal of Communication in Topology* volume *1,* Number *1,* 19-23 (2013).
3. Gyu Ihn Chae, K.K. Dube and O.S. Panwar., A note on weaky irresolute mappings, *Pusan Kyongnam Mathematical* Journal.Vol *1,* 89-100 (1985).
4. Lellis Thivagar. M. and Kavitha. J., On Binary structure of Supra Topological spaces, *Bol. Soc. Paran. Mat. (3s.) v.* 35 3 25–37 (2017).
5. Levine. N., Semiopen sets and semicontinuousmappings, Amer. Math. Monthly, 7044-46 (1963).
6. Maheshwari. S.N. and Thakur. S.S., On -Irresolute Mappings, Tamkang J. Math., Vol.*11,* No.2, 209-214 (1980).
7. Mashhour. A.S., Allam. A.A., Mahmoud. F.S. and Khedr. F.H., On Supra Topological Spaces., *Indian J. Pure appl. Math., 14(4),* 502-510 (1983)   .
8. Miguel Caldas Cueva, Weak and Strong forms of irresolute maps, *International J. Math and Math. Sci.* Vol *23*, No 4, 253-259 (2000).
9. Nithyanantha Jothi and Thangavelu. P., Topology between two sets., *Journal of Mathematical Sciences and Computer* Applications *1(3),* 95-107, 95-107 (2011).
10. Njastad. O., On some classes of nearly open sets, *Pacific J. Math.* 1961-970 (1965).