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Bianchi type-III String Cosmological Models in Brans-Dicke Theory of Gravity

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Abstract

A Bianchi type-III string cosmological models is investigated in scalar-tensor Brans-Dicke theory of gravity (Phys. Rev. 124, 925, 1961). For an exact solution of modified Einstein 's field equations (EFEs), we assumed time varying nature of deceleration parameter (DP) along with relation between scale factor and time, given by $a(t)=[t^\alpha e^t]^\frac{1}{n}$, α and n are non-negative constants. It is noticed from study that the power index has their own significance on the string cosmological models. It is also analyzed that the string tension density (λ) is an increasing function of time whereas the energy density (ρ) and the cosmological constant (Λ) are decreasing functions of time and converges to small value at present time.

Key words: Brans-Dicke Theory; Cosmological Constant; Variable Deceleration Parameter.

Mathematics subject classification 2010: 83F05.

Introduction

One of the most challenging problem in cosmology is to know the structure formulation of the universe. All the existing theories of gravitation are either based on amplification of quantum fluctuation in a scalar field ϕ during inflation or upon symmetry breaking phase transition in the early universe.

Among all the existing alternative theories, Brans-Dicke theory (BD)¹ of gravity is one of the most important scalar-theory due to its major role to address the various cosmological phenomena. According to BD

theory, the gravitational constant G has dynamic in nature, it vary with space and time. It also relate the gravitation constant G with scalar field ϕ (i.e. $\phi \cong G^{-1}$). The modified Einstein field equations for BD theory of gravity are expressed as

$$R_{ik} - \frac{1}{2} g_{ik} R = -\frac{8\pi}{c^4 \phi} T_{ik} - \frac{\omega}{\phi^2} \left(\phi_i \phi_k - \frac{1}{2} g_{ik} \phi^l \phi_l \right) - \frac{1}{\phi} (\phi_{;ik} - g_{ik} \phi) \quad (1)$$

Here wave operator is define by $\square = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2$, ∇ is Laplace's operator, R_{ik} is the Ricci curvature tensor, T_{ik} is the energy momentum tensor, R is the curvature scalar and ω is the Brans-Dicke dimensionless coupling constant. Also the law of conservation of momentum may be expressed as

$$T_{;k}^{ik} = 0. \quad (2)$$

Here 'semicolon' indicates co-variant derivative and 'comma' indicates partial derivative.

The variation in scalar field ϕ leads to the following equation:

$$2\phi \square \phi - \phi_k \phi^k = \frac{R}{\omega} \phi^2. \quad (3)$$

This latter equation can be simplified by substituting for R from the contracted form of Eq. (1) \rightarrow Eq. (3) can be simplified by substituting value of ω from Eq. (1). We finally get

$$\square \phi = \frac{8\pi}{(2\omega + 3)c^4} T, \quad (4)$$

Equation (4) leads to the anticipated scalar wave equation for scalar field ϕ with sources in matter. Because it contains a scalar field ϕ in addition to the metric tensor g_{ik} , therefore sometime BD theory is known as the scalar-tensor theory of gravitation.

Recent research findings²⁻⁵ indicate that viscosity play an important role in early stage evolution of the universe. Also it is well known that at early stage of the universe when neutrino decoupling occurred, the matter behaves like viscous fluid and coefficient of viscosity (ξ) decreases with time as universe expands. The viscous string cosmological models has been studied by several authors⁶⁻¹¹ in the context of general relativity, also many authors¹²⁻¹⁷ have discussed bulk viscous string cosmological models in BD theory. Very recently many cosmologists¹⁸⁻²⁰ have been investigated the Bianchi type-III viscous cosmic string cosmological models in BD theory.

Realizing the important of the topic in this study we have constructed bulk viscous string cosmological model with time dependent DP and cosmological constant in BD theory of gravity. Plan of this paper as follows; In section 2 metric and field equations governing the cosmological models are described, the exact solution of field equations is presented in section 3. In section 4 the physical and kinematic behavior of the models have been discussed. Finally, results and discussions are smmarized in last section 5.

Metric and Field Equations :

We may write spatially homogeneous and snisotropic Bianchi type-III space-time line element as

$$ds^2 = -dt^2 + R_1^2(t) dx^2 + e^{-2sx} R_2^2(t) dy^2 + R_3^2(t) dz^2. \quad (5)$$

Here potential R_1 , R_2 and R_3 are the functions of cosmic time t only and s is a constant.

The energy-momentum tensor T_{ik} for a cloud of strings in the presence of bulk viscous fluid containing one dimensional cosmic string is given by

$$T_{ik} = (\rho + \bar{p})u_i u_k + \bar{p} g_{ik} - \lambda v_i v_k, \quad (6)$$

where λ is a string tension density, \bar{p} is effective pressure, ρ is the proper energy density for cloud strings with particles attached to them, u_k is the four-velocity vector and x^k is a unit space-like vector along the direction of string. The vectors u^k and x^k satisfy the conditions $u_k u^k = 1 = -x_k x^k$, $u^k x_k = 0$.

$$\begin{aligned} T_{11} &= \bar{p} - \lambda, & T_{22} &= T_{33} = \bar{p}, & T_{44} &= \rho, \\ \Rightarrow T &= T_{11} + T_{22} + T_{33} + T_{44} = 3\bar{p} - \rho - \lambda. \end{aligned} \quad (7)$$

Above ρ , \bar{p} and λ are the functions of cosmic time 't' only. The particle density (ρ_p) of the configuration is given as

$$\rho = \rho_p + \lambda. \quad (8)$$

The string tension density λ , may takes positive or negative values. In literature Letelier, Berman and Som^{21,22} established that a negative value of λ represents the universe filled with no string, whereas positive value of λ indicate the universe filled with string particles. Here the effective pressure \bar{p} may be define as

$$\bar{p} = p - 3\xi H, \quad (9)$$

where ξ is the bulk viscosity coefficient and H Hubble parameter.

For the metric given in equation (5) the field equation (1) may be expressed as:

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} - \frac{s^2}{R_1^2} + \frac{\ddot{\phi}}{\phi} + \frac{\omega \dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} \right) = -8\pi\phi^{-1}(\bar{p} - \lambda) + \Lambda. \quad (10)$$

$$\frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1}{R_1} + \frac{\dot{R}_3 \dot{R}_1}{R_3 R_1} + \frac{\ddot{\phi}}{\phi} + \frac{\omega \dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_3}{R_3} \right) = -8\pi\phi^{-1}\bar{p} + \Lambda. \quad (11)$$

$$\frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{\ddot{\phi}}{\phi} + \frac{\omega \dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) = -8\pi\phi^{-1}\bar{p} + \Lambda. \quad (12)$$

$$\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{\dot{R}_3 \dot{R}_1}{R_3 R_1} - \frac{s^2}{R_1^2} - \frac{\omega \dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) = -8\pi\phi^{-1}\bar{p} + \Lambda. \quad (13)$$

$$\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2} = 0, \quad (14)$$

$$\ddot{\phi} + \dot{\phi} \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) = \frac{8\pi(3\bar{p} - \rho - \lambda)}{(3 + 2\omega)}, \quad (15)$$

here an over head dot denote derivatives with respect to cosmic time t.

We may introduce cosmological parameters such as the spatial volume (V), the expansion scalar (θ), the Hubble's parameter (H), the DP (q), the anisotropy parameter (A_m) and the shear scalar (σ) for the metric (5) defined as,

$$V = a^3 = R_1 R_2 R_3, \quad (16)$$

$$\theta = u_{;k}^k = 3H = H_1 + H_2 + H_3, \quad (17)$$

here $H_1 = \frac{\dot{R}_1}{R_1}$, $H_2 = \frac{\dot{R}_2}{R_2}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in directions of x, y and z axis respectively.

$$q = -\frac{a\ddot{a}}{a^2} = -\left(1 + \frac{\dot{H}}{H^2}\right). \quad (18)$$

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H}\right)^2, \quad (19)$$

$$\sigma^2 = \frac{1}{2} \left(\sum_{i=1}^3 H_i^2 - 3H^2\right), \quad (20)$$

Here, $\Delta H_i = H_i - H$, $i = 1, 2, 3$.

Now we required the solution of above stated field equations (10)-(15). This is presented in next section.

Solution of the Field Equations :

On integrating equation (14), we have

$$R_1 = l_0 R_2, \quad (21)$$

here l_0 is a constant of integration, it can be taken as unity to avoid further complication, Therefore

$$R_2 = R_1. \quad (22)$$

On putting $R_2 = R_1$ into the field equations (10)-(13) and (15), we get following set of field equations,

$$2\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_1^2}{R_1^2} - \frac{s^2}{R_1^2} + \frac{\ddot{\phi}}{\phi} + \frac{\omega\dot{\phi}^2}{2\phi^2} + 2\frac{\dot{R}_1}{R_1} \frac{\dot{\phi}}{\phi} = -8\pi\phi^{-1}(\bar{p} - \lambda) + \Lambda, \quad (23)$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1}{R_1} \frac{\dot{R}_3}{R_3} + \frac{\ddot{\phi}}{\phi} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_3}{R_3}\right) = -8\pi\phi^{-1}\bar{p} + \Lambda, \quad (24)$$

$$\frac{\dot{R}_1^2}{R_1^2} + 2\frac{\dot{R}_1\dot{R}_3}{R_1R_3} - \frac{s^2}{R_1^2} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi} \left(2\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_3}{R_3}\right) = -8\pi\phi^{-1}p + \Lambda, \quad (25)$$

$$\ddot{\phi} + \dot{\phi} \left(2\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_3}{R_3}\right) = \frac{8\pi(3\bar{p} - p - \lambda)}{(3 + 2\omega)}, \quad (26)$$

The set field equations (23)-(26) contains four independent equations along with seven unknown parameters R_1 , R_3 , λ , ρ , \bar{p} , ϕ and Λ . Therefore for an exact solution we need at least three more constraints related to these parameters. Hence, we applying following assumptions:

(i) It is assumed that the bulk viscosity function $\xi(t)$ is proportional to some power of energy density ρ (see^{23, 24}) i.e.

$$\xi \propto \rho^\beta, \tag{27}$$

$$\xi = \xi_0 \rho^\beta, \tag{28}$$

here ξ_0 is proportionality constant and $\beta \geq 0$ is a constant. Now with the help of equations (7), (24)-(26) we get,

$$-8\pi\phi^{-1}[\gamma\rho - \xi_0\rho^\beta\theta] + \Lambda = \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_3}{R_3}\right), \tag{29}$$

In this context we have discuss two cases i.e. $\beta = 0$ and $\beta = 1$.

Case 1: If $\beta = 0$, then $\xi = \xi_0$

$$-8\pi\phi^{-1}\gamma\rho + \Lambda = -8\pi\xi_0\theta\phi^{-1} + \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_3}{R_3}\right). \tag{30}$$

On subtracting equation (25) from equation (30), we have

$$8\pi\phi^{-1}(1-\gamma)\rho = -8\pi\phi^{-1}\xi_0\theta + \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\omega\dot{\phi}^2}{2\phi^2} - \frac{\dot{R}_1\dot{\phi}}{R_1\phi} - \left(\frac{\dot{R}_1}{R_1}\right)^2 + \frac{s^2}{R_1^2}, \tag{31}$$

On substituting the value of $8\pi\rho\phi^{-1}$ from equation(31) into equation(25), we have expression for cosmological constant Λ as

$$\begin{aligned} \Lambda(1-\gamma) = & -8\pi\phi^{-1}\xi_0\theta + \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\ddot{\phi}}{\phi} + (1-2\gamma)\frac{\dot{R}_1\dot{R}_3}{R_1R_3} + (1+\gamma)\frac{\omega\dot{\phi}^2}{2\phi^2} + (1-2\gamma)\frac{\dot{R}_1\dot{\phi}}{R_1\phi} \\ & - \gamma\frac{\dot{R}_1^2}{R_1^2} + (1-\gamma)\frac{\dot{R}_3\dot{\phi}}{R_3\phi} + \gamma\frac{s^2}{R_1^2} \end{aligned} \tag{32}$$

Case 2: If $\beta = 1$, then $\xi = \xi_0\rho$

$$8\pi\rho = \frac{\phi}{[\gamma - 1 - \xi_0\theta]} \left[\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{R}_1^2}{R_1^2} - \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\omega\dot{\phi}^2}{\phi^2} - \frac{\dot{R}_1\dot{\phi}}{R_1\phi} + \frac{s^2}{R_1^2} \right], \tag{33}$$

$$\begin{aligned} \Lambda = & \frac{1}{[\gamma - 1 - \xi_0\theta]} \left[\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\ddot{\phi}}{\phi} - \frac{\dot{R}_1^2}{R_1^2} - \frac{\dot{R}_1\dot{R}_3}{R_1R_3} + \frac{\omega\dot{\phi}^2}{\phi^2} - \frac{\dot{R}_1\dot{\phi}}{R_1\phi} + \frac{s^2}{R_1^2} \right] - \left(\frac{\dot{R}_1}{R_1}\right)^2 + 2\frac{\dot{R}_1\dot{R}_3}{R_1R_3} \\ & - \frac{s^2}{R_1^2} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi}\left(2\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_3}{R_3}\right). \end{aligned} \tag{34}$$

(ii) The DP q is taken as a function of cosmic time 't' i.e.

$$q = -\frac{a\ddot{a}}{a^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = q(t)(\text{say}), \tag{35}$$

as studied and published by our peer research group²⁵⁻²⁸, we may also believe that time dependence of the scale factor $a(t)$, reflect mainly in expansion rate of the universe. So under this motivation we have decided to find an exact solution of EFEs along with study of various others associated aspect with time varying DP. The general solution of equation (35) is given by the ansatz for scale factor in term of cosmic time as suggested by Amirhashchi *et al.* 2011²⁹ as

$$a(t) = [t^\alpha e^t]^\frac{1}{n}, \quad (36)$$

here α and n are non negative constants.

(iii) Collins *et al.*³⁰ suggested that the shear scalar σ is proportional to scalar expansion θ which we may write as

$$R_1 = R_3^m, \quad (37)$$

where m is a positive constant, which takes care of the anisotropy of the space.

(iv) Without any loss of generality, we shall assume that the scalar field ϕ is some power of the scale factor; i.e., the power law relation between scale factor a and scalar field ϕ has been suggested by Johri and Desikan (see³¹) may be expressed as,

$$\phi = \phi_0 [a(t)]^b, \quad (38)$$

where ϕ_0 is a proportionality constant and b is an arbitrary constant.

From equations (13), (27) and (28), we get the expression for the metric potential functions as

$$R_1 = R_2 = [t^\alpha e^t]^\frac{3m}{n(2m+1)}, \quad (39)$$

$$R_3 = [t^\alpha e^t]^\frac{3}{n(2m+1)} \quad (40)$$

$$H_1 = H_2 = \frac{3m}{n(2m+1)} \left[1 + \frac{\alpha}{t} \right] \quad (41)$$

$$H_3 = \frac{3}{n(2m+1)} \left[1 + \frac{\alpha}{t} \right], \quad (42)$$

After substituting the values of R_1 , R_2 and R_3 from equation (39) and (40), the metric (5) can be written as

$$ds^2 = - dt^2 + [t^\alpha e^t]^\frac{6m}{n(2m+1)} (dx^2 + e^{-2sx} dy^2) + [t^\alpha e^t]^\frac{6}{n(2m+1)} dz^2. \quad (43)$$

The metric (5) in term of redshift parameter z , may be expressed as

$$ds^2 = - dt^2 + [1 + z]^\frac{-6m}{2m+1} (dx^2 + e^{-2sx} dy^2) + [t^\alpha e^t]^\frac{-6}{2m+1} dz^2. \quad (44)$$

Physical and Kinematical Properties of the Models :

The some parameters such as spatial volume, expansion scalar (θ), Hubble parameter (H), DP (q), shear scalar (σ) and anisotropy parameter (A_m) are obtain by following mathematical expression:

$$V = ABC = [t^\alpha e^t]^\frac{3}{n}, \quad (45)$$

$$\theta = 3H = \frac{3}{n} \left[1 + \frac{\alpha}{t} \right] \tag{46}$$

$$q = -1 + \frac{n\alpha}{(t + \alpha)^2} \tag{47}$$

$$\sigma^2 = \frac{3(m-1)^2}{n^2(2m+1)^2} \left[1 + \frac{\alpha}{t} \right]^2, \tag{48}$$

$$A_m = 6 \left[\frac{(m+1)}{(2m+1)} \right]^2. \tag{49}$$

Above equation indicates A_m is constant throughout evolution of the universe.

Case 1: $\beta = 0$

$$8\pi\lambda = \frac{\phi_0 T_2^{\frac{b}{n}}}{n^2 k_1^2} \left[\{3(m-1)(6m+b+3)\} T_1^2 - 3\alpha n k_1 (m-1) \cdot \frac{1}{t^2} - s^2 n^2 k_1^2 \cdot T_2^{\frac{-6m}{nk_1}} \right], \tag{50}$$

where

$$T_1 = 1 + \frac{\alpha}{t}, T_2 = t^\alpha e^t, k_1 = 2m+1, k_2 = 9 + b^2 k_1^2 - 9m + \omega b^2 k_1^2 - 3b k_1 m, k_3 = \frac{b k_1}{3} + m + 1$$

$$8\pi\rho = \frac{24\pi\xi_0}{(\gamma-1)n} \cdot T_1 - \frac{\phi_0 T_2^{\frac{b}{n}}}{\gamma-1} \left[\frac{k_2}{n^2 k_1^2} \cdot T_1^2 - \frac{3\alpha k_3}{n k_1} \cdot \frac{1}{t^2} + s^2 T_2^{\frac{-6m}{nk_1}} \right], \tag{51}$$

$$8\pi\rho_p = \frac{24\pi\xi_0}{(\gamma-1)n} \cdot T_1 - \frac{\phi_0 T_2^{\frac{b}{n}}}{\gamma-1} \left[\frac{k_2 + (\gamma-1)(m-1)(6m+b+3)}{n^2 k_1^2} \cdot T_1^2 - \frac{3\alpha \{m + 1 \frac{b k_1}{3} - (\gamma-1)(m-1)\}}{n k_1} \cdot \frac{1}{t^2} + \gamma s^2 T_2^{\frac{-6m}{nk_1}} \right], \tag{52}$$

$$8\pi p = \frac{-24\pi\xi_0\gamma}{(\gamma-1)n} \cdot T_1 - \frac{\gamma\phi_0 T_2^{\frac{b}{n}}}{1-\gamma} \left[\frac{k_2}{n^2 k_1^2} \cdot T_1^2 - \frac{3\alpha k_3}{n k_1} \cdot \frac{1}{t^2} + s^2 T_2^{\frac{-6m}{nk_1}} \right], \tag{53}$$

$$\Lambda = -\frac{1}{(\gamma-1)} \left[\frac{9m^2(1-\gamma) + 3m(3-6\gamma+k_1-2\gamma b k_1) + \frac{b^2}{2}\omega k_1(1+\gamma)}{n^2 k_1^2} \cdot T_1^2 \right. \\ \left. + \frac{9k_1(1-\gamma) + b^2 k_1^2 + 9}{n^2 k_1^2} \cdot T_1^2 - \frac{3\alpha k_3}{n k_1} \cdot \frac{1}{t^2} + \gamma s^2 T_2^{\frac{-6m}{nk_1}} \right] + \frac{24\pi\xi_0 T_1 \phi_0^{-1} T_2^{\frac{-b}{n}}}{n(\gamma-1)}, \tag{54}$$

Case 2: $\beta = 1$

$$8\pi\rho = \frac{8\pi\xi}{\xi_0} = \frac{n\phi_0 T_2^{\frac{b}{n}}}{n(\gamma-1) - 3\xi_0 T_1} \left[\frac{k_2}{n^2 k_1^2} \cdot T_1^2 - \frac{3\alpha k_3}{nk_1} \cdot \frac{1}{t^2} + s^2 T_2^{\frac{-6m}{nk_1}} \right], \quad (55)$$

$$8\pi\rho_p = \frac{n\phi_0 T_2^{\frac{b}{n}}}{n(\gamma-1) - 3\xi_0 T_1} \left[\frac{k_2}{n^2 k_1^2} \cdot T_1^2 - \frac{3\alpha k_3}{nk_1} \cdot \frac{1}{t^2} + s^2 T_2^{\frac{-6m}{nk_1}} \right] - \frac{\phi_0 T_2^{\frac{b}{n}}}{n^2 k_1^2} \left[\{3(m-1)(6m+b+3)\} T_1^2 - 3\alpha nk_1(m-1) \cdot \frac{1}{t^2} - s^2 n^2 k_1^2 \cdot T_2^{\frac{-6m}{nk_1}} \right] \quad (56)$$

$$8\pi\rho = \frac{n\gamma\phi_0 T_2^{\frac{b}{n}}}{n(\gamma-1) - 3\xi_0 T_1} \left[\frac{k_2}{n^2 k_1^2} \cdot T_1^2 - \frac{3\alpha k_3}{nk_1} \cdot \frac{1}{t^2} + s^2 T_2^{\frac{-6m}{nk_1}} \right], \quad (57)$$

$$\Lambda = \frac{n}{n(\gamma-1) - 3\xi_0 T_1} \left[\frac{k_2}{n^2 k_1^2} \cdot T_1^2 - \frac{3\alpha k_3}{nk_1} \cdot \frac{1}{t^2} + s^2 T_2^{\frac{-6m}{nk_1}} \right]$$

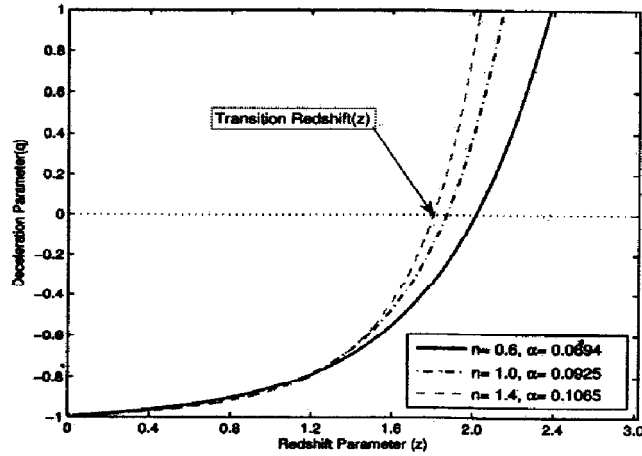


Figure 1. Plot of redshift z versus DP q .

$$+ \frac{9m(2m+1) - \frac{\omega b^2 k_1^2}{2} - 3b k_1^2}{n^2 k_1^2} \cdot T_1^2 - s^2 T_2^{\frac{-6m}{nk_1}} \quad (58)$$

We have presented the behavior of DP q with redshift parameter z in Fig.1, it is clear from concern figure that the q increasing with z and changing sign negative to positive for $n = 0.6, 1.0, n = 1.4$, which indicates the transition phase of the universe.

From Fig. 2, we observe that the string tension density λ is an increasing function of time, which

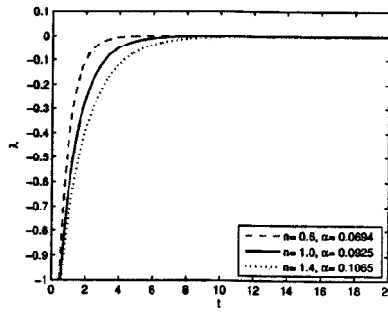


Figure 2. Plot of string tension density λ versus time t . For $b = -1$, $\omega = 1$, $\phi_0 = 1$.

is always negative and approaches to zero at late time. As suggested by Letelier (1979) [21] the string tension density λ may have positive or negative values, corresponding to $\lambda > 0$ the string dominant over particle whereas in case of $\lambda < 0$ the string disappear from universe. In our case the particles density dominate over the string tension density at present epoch. It is self explanatory from Fig. 3 and Fig. 4 the energy density ρ and particle density ρ_p are decreasing function of cosmic time for both cases $\beta = 0$ and $\beta = 1$.

In Fig. 5 we have plotted cosmological constant Λ with cosmic time t . It may be seen from figure that cosmological constant Λ is decreasing function of time and approaches to small value at late time. This type of behavior of Λ is good agreement with recent cosmic observations.

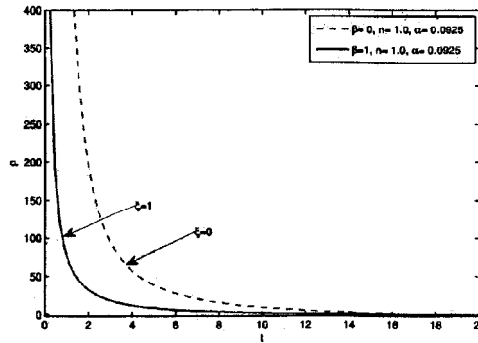


Figure 3. Plot of energy density ρ versus time t . For $\beta = 0, 1$, $b = -1$, $\omega = 1$, $\phi_0 = \xi_0 = 1$

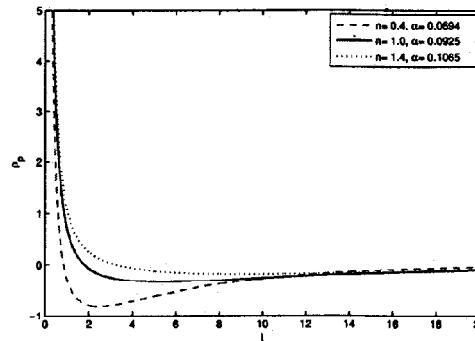


Figure 4. Plot of particle density ρ_p versus time t . For $\beta = 0, 1$, $b = -1$, $\omega = 1$, $\phi_0 = \xi_0 = 1$

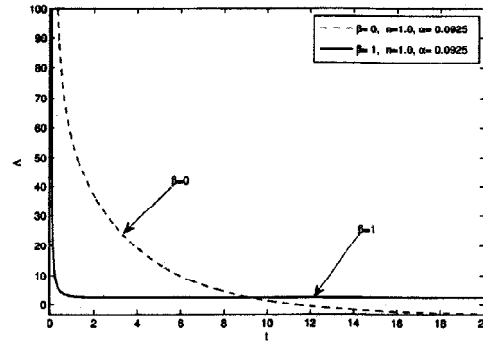


Figure 5. Plot of cosmological constant Λ versus time t . For $\beta = 0$, $b = -1$, $\omega = 1$, $\phi_0 = \xi_0 = 1$.

Concluding Remarks

As discussed in foregoing sections we have presented the spatially homogeneous and anisotropic Bianchi- III space-time cosmological model with time dependent DP q and dynamical cosmological constant Λ in the frame work of BD theory of gravity with additional assumption $(a) = [t^\alpha e^t]^\frac{1}{n}$, where n and α are positive constants. We have presented a class of models with different choices of n and β . The main conclusion of the model are presented below:

- The current study show that the universe starts evolving from zero volume at $t = 0$ (see eq(45)) and thereafter expanding continuously from early decelerating phase to present accelerating phase. As $t \rightarrow 0$, the expansion scalar $\theta \rightarrow \infty$, which indicates the early inflationary phase of the universe. Therefore we can say that the universe grow up from Big-Bang.
- The universe show acceleration behavior for $n \leq 1$ and transition phase i.e. early deceleration phase to current acceleration phase for $n > 1$. This type nature of universe show the signature flipping.
- It is observed form Fig. 2 and Fig. 3 for both cases $\beta = 0$ and $\beta = 1$, which indicates that the string tension density is negative time whereas particle density ρ_p is positive at early time and at late time both are converges to zero. Hence, the string disappears from universe at present time.
- The cosmological constant Λ is a decreasing function of time and it converges to a small positive value at late time (See Fig. 5). This type of behavior of cosmological constant Λ is supported by recent observations data.

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