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Section A

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Analysis of Electrical Networks : Graph Theoretic Approach

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Abstract

Graph theory, a branch of Mathematics plays a vital rule in studying interdisciplinary subjects such as physics, Chemistry, Engineering etc. Study of the properties of electrical circuits with the help of graph theory is a growing trend in mathematical and electrical fields. Electrical circuits consists of nodes and branches which obeys Kirchhoff 's current laws, Kirchhoff 's voltage laws etc. There are various well known theorems such as Norton's theorem, Thevenin's theorem, Superposition theorem, Millman Theorem etc for network analysis. In this paper, we try to analyze Millman's theorem with the help of Graph theorem.

Key words : Graph theory, Electrical circuits, Branch current, Loop current,

2000 A.M.S. subject classification: 05C50, 11F27.

1. Introduction

The electrical circuits[19] are the main building blocks of electrical appliances. Without the electrical appliances today's civilization become fully paralyzed. Now-a-days Graph theory[1,2], a branch of Mathematics has impact on almost every other branch. In engineering mathematics also, graph theory plays a vital rule. Study of the properties of electrical circuits[14] with the help of graph theory is a growing trend in mathematical and electrical fields. Electrical circuits consists of nodes[3] and branches[3] which obeys Kirchhoff's current laws, Kirchhoff's voltage law[12] etc. The scientists now-a-days engage themselves to construct simple cost effective circuits. There are various well known theorems such as Norton's theorem, Thevenin's theorem[21], Superposition theorem[12], Millman's Theorem[20,21] etc to construct simple circuits from the complicated circuits. In the graph of the circuits, all the elements of the circuits are replaced by lines with circles or dots at both ends. The lines represent branches and the dots represent nodes of the circuits. From these graphs, we can find matrices related to them by different rules. In our work we try to study electrical networks[9,10,16,17], and its simplified circuits and their associated currents and voltages through network equilibrium[11] equations which obey the laws of electricity.

1.1 Graph: A graph is a set of ordered pair $G = (V, E)$ of sets where $E = \{\{x, y\} : x, y \in V\}$. The elements of V are called vertices (or nodes) of the graph G and the elements of E are called edges. So in a graph a vertex set is a set of points $\{x_1, x_2, x_3 \dots . x_n\}$ and edge is a line which connects two points x_i *and* x_j. Such graphs are called undirected graphs. A directed graph is similar to an undirected graph except the edge set $E \in V \times V$.

1.2 Electrical Circuits : An electrical circuit consists of internally connected elements viz resistors, capacitors, inductors, diodes, transistors etc. The behaviour of an electrical circuits generally depends upon two factors.

a) The characteristic of each of internally connected elements
b) The rule by which they are connected together.

The second factor gives a relation between electrical circuits with graph theory. A two terminal electrical element can be represented by an edge e_k. Associate with each edge there are two variables $V_k(t)$ and $i_k(t)$. The variable $V_k(t)$ is called the edge voltage and may be regarded as cross variable because it exist across the two end points. The other variable is called edge current and may be regarded as through variable because it flows through the edge. These variables must also obey the two laws of Kirchhoff's.

1.3 Kirchhoff's Current Law (KCL): For any lumped electrical network, at any time the net sum (taking into account the orientations) of all the currents leaving any node or vertex is zero. That is at rth vertex of the corresponding digraph, we must have

$$\sum_{k=1}^{e} a_{rk}\, i_k(t) = 0$$

Where a_{rk} is the rk th entry of the incidence matrix A of G and $i_k(t)$ is the amount of current flowing through the kth edge of G.

1.4 Kirchhoff's Voltage Law (KVL) : For any lumped electrical network, at any time the net sum (taking into account the orientations) of the voltages around a loop (i.e. circuit) is zero. In terms of the corresponding digraph, for the rth circuit we must have

$$\sum_{k=1}^{e} b_{rk} v_k(t) = 0$$

Where b_{rk} is the rk th entry of the circuit matrix B of G and $V_k(t)$ is the amount of voltage across the kth edge.

1.5 From Circuit to Graph :

A graph can be obtained from a circuit. We identify the graph G= (V, E) where V is the set of vertices and E is the set of edges. The edge between i[th] and j[th] vertices can be denoted by {i,j} ignoring the direction. Similarly the notation (i,j) can be used for oriented edges, where i is the start vertex and j is the end vertex. For example consider the circuit and its graph in the figure. There are five vertices and seven edges in the graph obtained from the given circuit. An edge is sometimes called branch and a vertex is called node in case of electrical circuits.

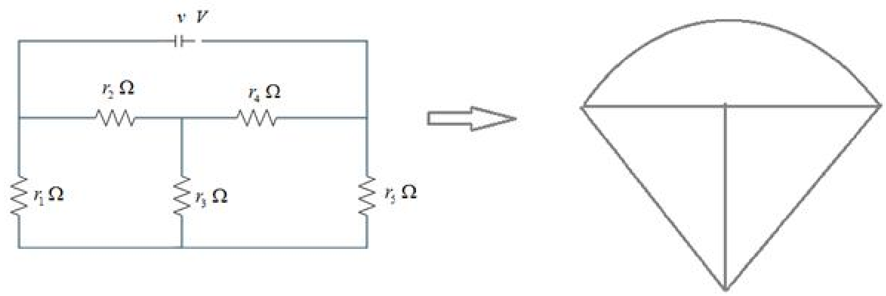


Fig:1. Circuit and its corresponding graph

1.6 Matrices Associated To A Graph :

1.6.1 . Fundamental Tie set matrix (Fundamental loop matrix) :

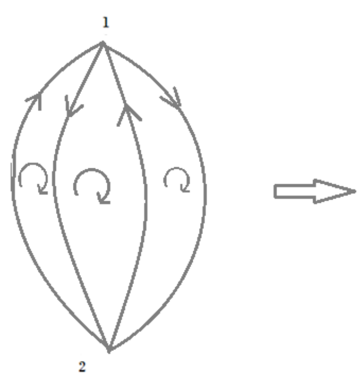
This matrix is associated to a fundamental loop i.e. a loop formed by only one link (branch that does not belong to a particular tree) associated with other twigs (branch of trees). Here we assume that the direction of loop currents and direction of the link is same.

So in the matrix

$$b_{ij} = 1, \textit{if the branch } b_j \textit{ in the fundamental loop } i \textit{ ane their reference direction oriented same}$$

$$b_{ij} = -1 \ \textit{if the branch } b_j \textit{ in the fundamental loop } i \textit{ ane their reference direction oriented opposite}$$

$$b_{ij} = 0 \ \textit{if the branch } b_j \textit{ is not in the fundamental loop } i$$



$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Fig: 2. Graph with its Incidence Matrix

1.6.2. Branch Impedance matrix $[Z_b]$:

It is a square matrix of order m where m is the no of branches having branch impedance as the diagonal elements and mutual impedance as off diagonal elements. If there is no transformer or mutual sharing then off diagonal entries are zero.

2. Review of Existing Literature:

Network analysis is important branch in electrical engineering. It was successfully studied [19] from nineteenth century by German scientist Hermann von Helmholtz in 1853 and by Leon Charles Thevenin (1857–1926) an electrical engineer in 1883. Hans Ferdinand Mayer (1895–1980) in 1926 and Bell Labs engineer Edward Lawry Norton (1898–1983) also studied[3] about the electrical networks. There are various network theorems such as Thevenin's theorem, Norton's Theorem, Millman's theorem, Superposition theorem, maximum power transfer theorem[3] etc. We studied all these theorems that these and it was observed that network theorems are little bit laborious and followed iterative method.

Therefore, we try to analyze the electrical networks with the help of network equilibrium equation of graph theory and try to overcome the limitations of the theorems with the following objectives:

3. Objectives of Study :

During literature survey it was found that there are many limitations in the existing theorems in network analysis. Therefore, we proposed to analyze the electrical networks with the help of network equilibrium equation of graph theory and overcome the limitations of the theorems with the following objectives such as:

(i) Study of electrical networks in details[3,8,12,13].
(ii) Study of graph theory[4,6] in details.
(iii) Establish the relation between graphs and networks.
(iv) Find out the limitations of the existing theorems in network analysis.
(v) Develop graph theoretic model to reduce the limitations of the theorems.
(vi) Formulation of computer programming for graph theoretic model.

4. Methodology :

Following the objectives first we have studied the laws of electrical circuits [18] and network theorems. Then we studied the basic rules of graph theory which relates the electrical circuits to matrices and graphs. Finally we successfully applied the network equilibrium equation of graph theory in network analysis to fulfill our objectives.

5. Millman's Theorem :

With the help of this theorem any numbers of parallel voltage sources can be reduce to one equivalent source.

Let us consider a number of parallel voltage sources $V_1, V_2, V_3, ..., V_n$ having internal resistances $R_1, R_2, R_3, ..., R_n$ respectively. The arrangement can be replaced by a single equivalent voltage source V in series with an equivalent series resistance R as shown in the figure .

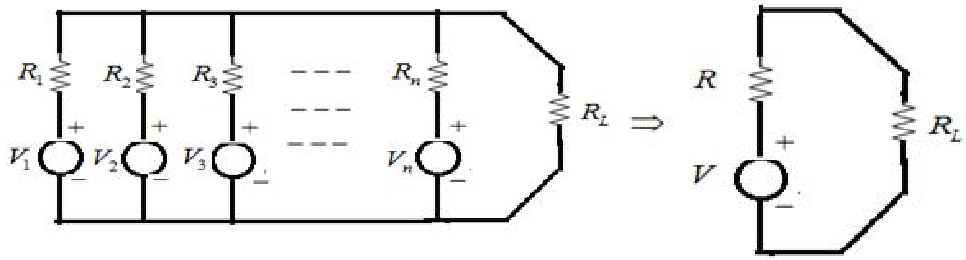


Fig: 3. Circuit with n-loops and its simplified circuit

By Millman's theorem the equivalent voltage and resistance of the reduced circuit is given by the relations

$$V = \frac{\pm V_1G_1 \pm V_2G_2 \pm V_3G_3 \pm ... \pm V_nG_n}{G_1 + G_2 + G_3 + ... + G_n} \text{ where } R = \frac{1}{G} = \frac{1}{G_1 + G_2 + G_3 + ... + G_n} .$$

Let us convert the voltage sources into current sources as follows

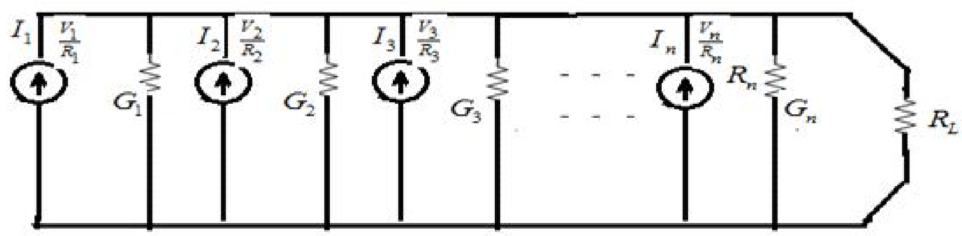


Fig: 4. Circuit havinh current sources

If I be the resultant current of the parallel current sources where G is the equivalent conductance.

$I = I_1 + I_2 + I_3 + ... + I_n$ and $G = G_1 + G_2 + G_3 + ... + G_n$.

Then the equivalent circuit will be

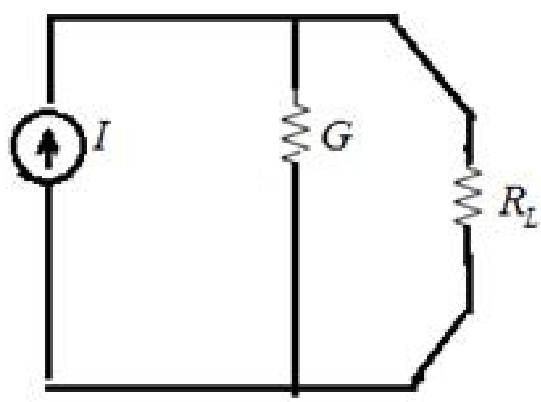


Fig: 5. Equivalent circuit of n-loops circuit

Now converting the current sources to equivalent voltage sources as

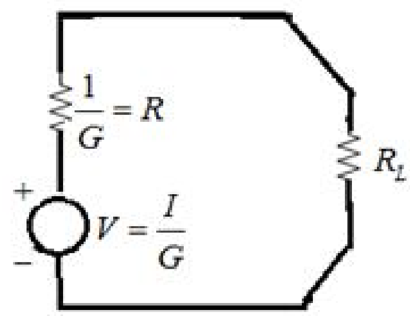


Fig: 6. Single loop circuit

Thus we have $V = \frac{I}{G} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm ..... \pm I_n}{G_1 + G_2 + G_3 + ... + G_n}$

Also $R = \frac{1}{G} = \frac{1}{G_1 + G_2 + G_3 + ... + G_n}$

and $V = \frac{\pm \frac{V_1}{R_1} \pm \frac{V_2}{R_2} \pm \frac{V_3}{R_3} \pm ..... \pm \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}}$

Where R is the equivalent resistance connected with the equivalent voltage source in series.

6. Our problem: With the help of Millman's theorem find the current through $r^4\ \Omega$ of the following circuit.

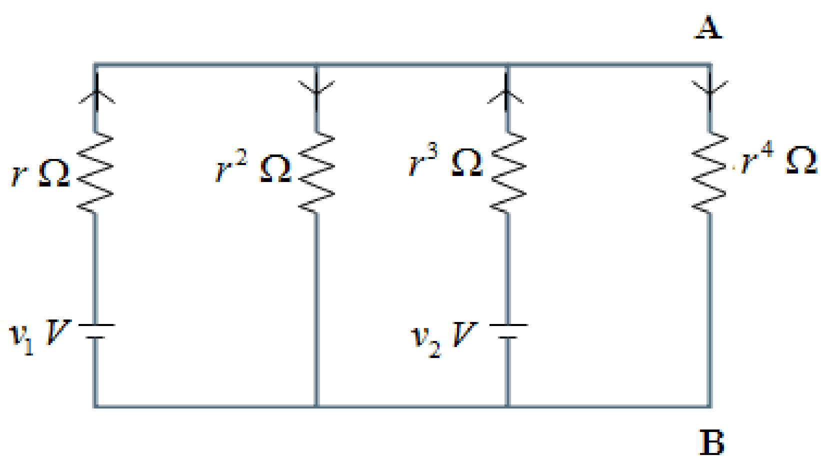


Fig: 7. Circuit having arbitrary voltage and resistances are in A.P.

Voltage across the terminal = $A-B=V_{AB}=\dfrac{\dfrac{E_1}{R_1}+\dfrac{E_2}{R_2}+\dfrac{E_3}{R_3}}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}=\dfrac{\dfrac{v_1}{r}+\dfrac{0}{r^2}+\dfrac{v_2}{r^3}}{\dfrac{1}{r}+\dfrac{1}{r^2}+\dfrac{1}{r^3}}=\dfrac{r^2v_1+v_2}{1+r+r^2}$

And equivalent resistance $R_{eq}=\dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}=\dfrac{1}{\dfrac{1}{r}+\dfrac{1}{r^2}+\dfrac{1}{r^3}}=\dfrac{r^3}{1+r+r^2}\,ohms$

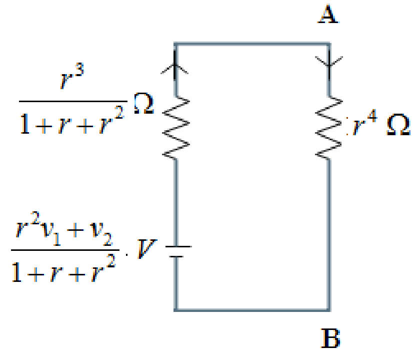


Fig: 8 Simplified circuit

Therefore by Millman's theorem the current through $r^4\ \Omega$ is obtained as

$$I_{14\Omega}=\frac{V_{Th}}{R_{eq}+r^4}=\frac{\dfrac{r^2v_1+v_2}{1+r+r^2}}{\dfrac{r^3}{1+r+r^2}+r^4}=\frac{r^2v_1+v_2}{r^3\left(1+r+r^2+r^3\right)}A$$

Now let us study this circuit with the help of graph theory.

The graph of the given circuit is as given below. This graph has two vertices 1 and 2 and five edges.

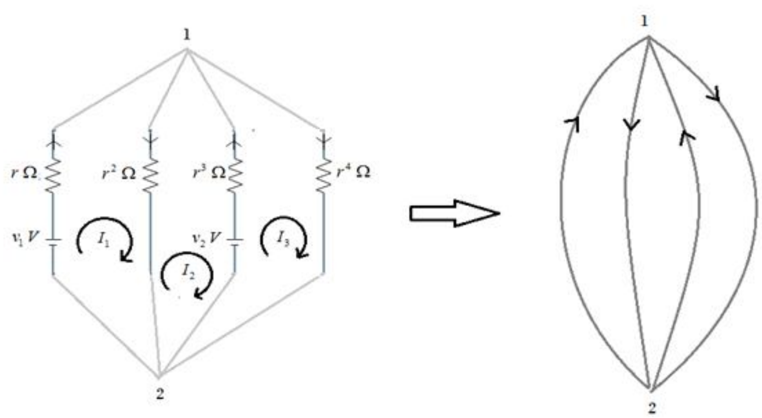


Fig : 9. Graph of the circuit

$$[B]=\begin{bmatrix}1&1&0&0\\0&-1&-1&0\\0&0&1&1\end{bmatrix}\qquad [V_s]=\begin{bmatrix}v_1\\0\\v_2\\0\end{bmatrix}\qquad [Z_b]=\begin{bmatrix}r&0&0&0\\0&r^2&0&0\\0&0&r^3&0\\0&0&0&r^4\end{bmatrix}$$

$$[B][V_s]=\begin{bmatrix}1&1&0&0\\0&-1&-1&0\\0&0&1&1\end{bmatrix}\begin{bmatrix}v_1\\0\\v_2\\0\end{bmatrix}=\begin{bmatrix}v_1\\-v_2\\v_2\end{bmatrix}$$

$$[B][Z_b]=\begin{bmatrix}1&1&0&0\\0&-1&-1&0\\0&0&1&1\end{bmatrix}\begin{bmatrix}r&0&0&0\\0&r^2&0&0\\0&0&r^3&0\\0&0&0&r^4\end{bmatrix}=\begin{bmatrix}r&r^2&0&0\\0&-r^2&-r^3&0\\0&0&r^3&r^4\end{bmatrix}$$

$$[B][Z_b][B']=\begin{bmatrix}r&r^2&0&0\\0&-r^2&-r^3&0\\0&0&r^3&r^4\end{bmatrix}\begin{bmatrix}1&0&0\\1&-1&0\\0&-1&1\\0&0&1\end{bmatrix}=\begin{bmatrix}r+r^2&-r^2&0\\-r^2&r^2+r^3&-r^3\\0&-r^3&r^3+r^4\end{bmatrix}$$

By Kirchhoff ‘s Voltage Law we have

$$[B][Z_b][B'][I]=-[B][V_s]$$

$$\Rightarrow\begin{bmatrix}r+r^2&-r^2&0\\-r^2&r^2+r^3&-r^3\\0&-r^3&r^3+r^4\end{bmatrix}\begin{bmatrix}I_1\\I_2\\I_3\end{bmatrix}=-\begin{bmatrix}v_1\\-v_2\\v_2\end{bmatrix}$$

$$\therefore\left(r+r^2\right)I_1-r^2I_2=-v_1$$

$$-r^2I_1+\left(r^2+r^3\right)I_2-r^3I_3=v_2$$

$$-r^3I_2+\left(r^3+r^4\right)I_3=-v_2$$

$$\therefore \quad I_1 = -\frac{r^2 v_1 + r v_1 + v_1 - r v_2}{r(1+r+r^2+r^3)} A \quad , \quad I_2 = -\frac{v_1 - v_2}{r\left(1+r^2\right)} A \quad , \quad I_3 = -\frac{r^2 v_1 + v_2}{r^3\left(1+r+r^2+r^3\right)} A$$

Now we have to find the branch current of the circuit

$$\left[i_b\right] = \left[B'\right]\left[I_L\right]$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\dfrac{r^2 v_1 + r v_1 + v_1 - r v_2}{r(1+r+r^2+r^3)} \\ -\dfrac{v_1 - v_2}{r\left(1+r^2\right)} \\ -\dfrac{r^2 v_1 + v_2}{r^3\left(1+r+r^2+r^3\right)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -\dfrac{r^2 v_1 + r v_1 + v_1 - r v_2}{r(1+r+r^2+r^3)} \\ -\dfrac{r^2 v_1 + v_2}{r(1+r+r^2+r^3)} \\ \dfrac{r^3 v_1 - r^3 v_2 - r^2 v_2 - v_2}{r^3(1+r+r^2+r^3)} \\ -\dfrac{r^2 v_1 + v_2}{r^3(1+r+r^2+r^3)} \end{bmatrix}$$

$$\therefore i_1 = -\frac{r^2 v_1 + r v_1 + v_1 - r v_2}{r(1+r+r^2+r^3)}, \; i_2 = -\frac{r^2 v_1 + v_2}{r(1+r+r^2+r^3)}, \; i_3 = \frac{r^3 v_1 - r^3 v_2 - r^2 v_2 - v_2}{r^3(1+r+r^2+r^3)}, \; i_4 = -\frac{r^2 v_1 + v_2}{r^3(1+r+r^2+r^3)}$$

Therefore current across the resistance $r^4\Omega$ is $i_4 = -\dfrac{r^2 v_1 + v_2}{r^3(1+r+r^2+r^3)}$, which is same as the current obtain by Millman's theorem. (Negative sign shows that orientation of the current is in opposite direction).

7. Conclusion

In view of above from our study it is clear that graph theoretic approach is the best effective method for network analysis or alternatively we can conclude that Millman's theorems can be replaced by graph theoretic model**.** In further study one can study the other electrical theorems using this graph theoretic model.

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