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A Note on Fuzzy Pairwise R_0 Bitopological Spaces

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Abstract

The purpose of this paper is to introduce the notion of fuzzy pairwise R_0 axiom in fuzzy bitopological spaces and study some of its properties. Several interesting results have been obtained viz. it satisfy hereditary, good extension, productive and projective properties.

Key words: Fuzzy R_0 topological space, Fuzzy pairwise R_0 bitopological spaces, Hereditary property, Good extension property, Productive and projective properties.

1. Introduction

Fuzzy R_0 spaces have been introduced and studied earlier by Hutton and Reilly[2] and Srivastava *et al.*[6] independently. We follow here the definition given by Srivastava *et al.*[6]. We introduce it as a generalization of F-R_0 spaces. We see that FP-R_0 spaces satisfy hereditary, productive and projective properties. We have also seen that it is good extension of P-R_0 in bitopological spaces.

2.Preliminaries :

Here we shall follow Lowen's definition of a fuzzy topological spaces (in short, an fts). The symbol 'I' will denote the unit interval [0,1] and I^X denotes the set of all fuzzy sets in X. All other undefined concepts are taken from[5,4,7]

Definition 2.1[5]. A triple (X, τ_1, τ_2), where X is a non empty set and τ_1, τ_2 are arbityrary fuzzy topologies on X, is called a fuzzy bitopological space (in short, fbts).

Let (X, τ_1, τ_2) be an fbts, $A \subseteq X$ and $\tau_{iA} = \{\lambda \mid A: \lambda \in \tau_i\}$ denote the subspace fuzzy topology[7] on A induced by $\tau_i, i = 1, 2,$ then $(A, \tau_{1A,} \tau_{2A})$ is called the subspace of (X, τ_1, τ_2) with the underlying set A.

Definition 2.2[7]. Let $\{(X_i, \tau_{1i}, \tau_{2i}) : i \in A\}$ be a family of fbts'. Then the space $(\Pi X_i, \Pi\tau_{1i}, \Pi\tau_{2i})$ is called their product where $\Pi\tau_{ki}$ denotes the usual product fuzzy topology of the family $\{\tau_{ki}: i \in A\}$ of fuzzy topologies on X, k=1, 2.

A property P is called productive if the product fbts (X_i, r_i $\Pi\tau_j$) has P if each coordinate fuzzy space has P. A property P is called projective if the product fbts (X_i, r_i $\Pi\tau_j$) has P implies that each coordinate fuzzy space has P.

A property P of an fbts (X,τ_i,τ_j) is said to be hereditary if every subspace of the space possesses P.

The ‘good extension’ property in the sense of Lowen[5] has been extended to the case of fuzzy bitopological spaces as follows:

A fuzzy bitopological analogue FP of a bitopological property P is said to be a good extension of P if for every bitopological space (X,T_1, T_2) possesses P iff the fbts (X, ω (T_1), ω (T_2)) possesses FP.

3. Fuzzy Pairwise R_0 bitopological spaces :

In this section, we introduce the concept of fuzzy R_0 bitopological spaces and study some of its properties.

Definition 3.1 : A bitopological space (X,T_1,T_2) is pairwise R_0 (in short, P-R_0) iff for all $x,y\in X,x\neq y$ whenever there is a $U\in T_1$ such that U(x)=1, U(y)=0 there is also $V\in T_2$ such that V(y)=1,V(x)=0.

Definition 3.2: A fuzzy bitopological space (X,τ_1,τ_2) is fuzzy pairwise R_0 (in short, FP-R_0) iff for all x, $y\in X$, $x\neq y$ whenever there is a $U\in\tau_1$, such that U(x)=1,U(y)=0 there is also $V\in\tau_2$ such that V(y)=1, V(x)=0.

The definition of fuzzy pairwise R_0 bitopological spaces are good extension of pairwise R_0 as seen below:

Theorem 3.1: A bitopological space (X,T_1,T_2) is P-R_0 iff the fuzzy bitopological spaces (X,ω (T_1),ω(T_2)) is FP-R_0.

Proof: Let us assume that (X,T_1,T_2) be P-R_0. Take $x,y\in X$, $x\neq y$. Suppose $\exists$ $U\in$ ω(T_1) such that U(x)=1, U(y)=0. Then $U^{-1}(0,1]\in T_1$ and is such that $x\in U^{-1}(0,1]$, $y\notin U^{-1}(0,1]$. Now since (X,T_1,T_2) is R_0 there is $V\in T_2$ such that $x\notin V$, $y\in V$. Now considering $V\in$ω(T) we found that

V(y)=1, V(x)=0. Hence (X,T_1,T_2) is fuzzy pairwise R_0.

Conversely let (X,(T_1),ω(T_2)) be fuzzy pairwise R_0. Take $x,y\in X$

$x\neq y$ Then whenever $\exists U\in T_1$ such that $x\in U and\ y\notin U\ then\ the\ crisp$ fuzzy open set U is such that U(x)=1,

U(y)=0 and since (X,ω(T_1),ω(T_2)) is fuzzy R_0 there is $V\in$ ω(T)

Such that V(x)=0, V(y)=1. Now $V^{-1}(0,1]\in T_2$ such that $x\notin V^{-1}(0,1]$

and $y\in V^{-1}(0,1]$ implying that (X,T_1,T_2) is pairwise R_0.

Next we show that FP-R_0 satisfy hereditary property as:

Theorem 3.2: Every subspace of fuzzy pairwise R_0 fuzzy bitopological space is also fuzzy pairwise R_0.

Proof : Let the fbts (X,τ_1,τ_2) be a fuzzy pairwise R_0. Let (Y,τ_{1Y},τ_{2Y}) be its subspace. Let x , $y\in Y\subseteq X,x\neq y$. Since ($X,\tau_1,\tau_2$) is fuzzy pairwise R_0 whenever there is a $U\in\tau_1$ such that U(x)=1,

U(y)=0 there is also $V\in\tau_2$ such that V(y)=1, V(x)=0. Now we see that whenever there is a $U_Y(x)=U\cap Y(x)=1$,

$U_y(y)=U\cap Y(y)=0$ there is also $V\in\tau_2$ such that $V_Y(y)=V\cap Y(y)=1$,

$V_Y(x)=V\cap Y(x)=0$. This implying that (Y,τ_{1y},τ_{2y}) is also fuzzy pairwise R_0.

Proposition 3.1: An fbts (X,τ_1,τ_2) is fuzzy pairwise R_0 iff the fts (X,τ_1) and (X,τ_2) are fuzzy R_0.

Proof: First let us suppose that the fbts (X,τ_1,τ_2) is fuzzy pairwise R_0. Then for $x,y\in$ X, $x\neq y$ whenever there is a $U_1\in\tau_1$ such that

$U_1(x)=1$, $U_1(y)=0$ there is also $V_1(y)=1$ and $V_1(x)=0$. Now if we take $y,x\in X$, $x\neq y$ then whenever there is a $U_2\in\tau_1$ such that $U_2(x)=0$,

$U_2(y)=1$ there is also $V_2\in\tau_2$ such that $V_2(y)=1$, $V_2(x)=0$. Thus for $x,y\in X$ $x\neq y$ we have seen that whenever there is τ_1-fuzzy sets U_1 and U_2 such that $U_1(X)=1$, $U_1(y)=0$ and $U_2(x)=0$, $U_2(y)=1$ there is τ_2-fuzzy open set V_1 and V_2 such that $V_1(x)=0$,$V_1(y)=1$ and $V_2(x)=1$, $V_2(y)=0$. Thus (X,τ_1) and (X,τ_2) is fuzzy R_0.

Conversely suppose that (X,τ_1) and (X,τ_2) is fuzzy R_0. Then first taking (X,τ_1) fuzzy R_0, for $x,y\in$ X , $x\neq y$ whenever there is a $U\in\tau_1$ such that U(x)=1, U(y)=0 there is also $V\in\tau_1$ such that V(x)=0 ,V(y)=1. Next taking (X,τ_2) fuzzy

R_0, for $y,x\in X$, $y\neq x$ whenever there is $U\in\tau_2$ such that $U(y)=1$, $U(x)=0$ there is a $V\in\tau_2$ such that $V(x)=0$, $V(y)=1$. Thus for $x,y\in X$, $x\neq y$ whenever there is $U\in\tau_1$ such that $U(x)=1$, $U(y)=0$ there is also $V\in\tau_2$ such that $V(x)=0,V(y)=1$

Implying that (X,τ_1,τ_2) is fuzzy pairwise R_0.

Next we show that FP-R_0 satisfies productive and projective properties:

Theorem 3.3: Let $\{(X_i,\tau_{1i},\tau_{2i});\ i\in A\}$ be a family of fbts. Then the product fbts $(\Pi X_i,\Pi\tau_{1i},\Pi\tau_{2i})$ is FP-R_0 iff each coordinate space $(X_i,\tau_{1i},\tau_{2i})$ is FP-R_0.

Proof: First let each coordinate space $\{(X_i,\tau_{1i},\tau_{2i}),i\in A\}$ be FP-R_0. Then show that the product fbts is FP-R_0, let $x,y\in X$, $x\neq y$, Let

$x=\Pi\ x_i$, $y=\Pi y_i$ where $x_j\neq y_j$ for some $j\in A$. Now take $x_j,y_j\in X_j$. Since $(X_j,\tau_{1j},\tau_{2j})$ is FP-R_0 whenever there is a $U_j\in\tau_{1j}$ such that $U_j(x_j)=1$, $U_j(y_j)=0$ there is a $V_j\in\tau_{2j}$ such that $V_j(y_j)=1$, $V_j(x_j)=0$. Now consider $U=\Pi U_i'$ and $V=\Pi V_i'$ where $U_i'=V_i'$ for $i\neq j$,$U_j'=U_j$ and $V_j'=V_j$ then whenever there is a $U\in\Pi\tau_{1j}$ such that $U(x)=1$, $U(y)=0$ there is a $V\in\tau_{2j}$ such that

$V(y)=1$, $V(x)=0$. Thus the product fuzzy bitopological space is FP-R_0.

Conversely, let $(\Pi X_i,\Pi\tau_{1i},\Pi\tau_{2i})$ be fuzzy pairwise R_0. Consider any coordinate spaces say $(X_i,\tau_{1i},\tau_{2i})$ choose x_i, $y_i\in X_i$, $x_i\neq y_i$. Construct $x,y\in X$ such that $x=\Pi x_j'$, $y=\Pi y_j'$ where

$›\Rightarrow r<\inf U_j^r(x_j')$ for all $j\in A\Rightarrow r<U_j^r(x_j')$ for all $j\in A\Rightarrow U(y)=0\Rightarrow \Pi U_j(y)=0$. Then there is a fuzzy point $y_s\in V$ such that there exist a basic fuzzy open set $\Pi V_j^s\in\Pi\tau_{2j}$ such that $y_s\in\Pi V_j^s\subseteq V$

$\Rightarrow s<V_j^s(y_j)$ for all $j\in A$ and $\Pi V_j^s(y)=0$.

Now $\Pi U_j^r(y)=0\Rightarrow U_j^r(y_j)=0$. Hence $U_j^r(y_j)=U_j^r(x_j)›\ r$

Similarily $\Pi V_j^s(y)=0\Rightarrow V_j^s(x_i)=0$. Thus $V_i^s(x_i)=V_i^s(y_i)›\ s$

Consider Sup $U_i^r=U_i\in\tau_{1i}$ and Sup $V_i^s=V_i\in\tau_{2i}$ whenever there is a

$U_i(x_i)=1$, $U_i(y_i)=0$ there is a $V_i\in\tau_{2j}$ such that $V_i(y_i)=1$, $V_i(x_i)=0$

Implying that $(X_i,\tau_{1i},\tau_{2i})$ is fuzzy pairwise R_0.

Conclusion

In this paper the concept of fuzzy pairwise R_0 axiom in a fuzzy bitopological spaces has been introduced. The appropriateness of our definition is established by proving some interesting relevant results. We proved that FP-R_0 in fuzzy bitopological spaces is good extension of the corresponding concept of P-R_0 in bitopological spaces. Our definition of FP-R_0 also satisfy hereditary, productive and projective properties.

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